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Introduction to Control Engineering

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Contents

Chapter 1

Introduction

The learning objectives of this chapter:

- Appreciate the role and importance of control systems in our daily lives.
- Understand the basic components of a control system.
- Understand the difference between the open-loop and closed-loop systems, and the role of feedback in a closed-loop control system.
- Gain a practical sense of real-life control problems.

1.1 What is control?

According to Collins English Dictionary, as a verb, to control a piece of equipment, process, or system means to make it work in the way that you want it to work. What physical variables can be controlled? They could be position, speed, acceleration, or torque in vehicle control problems. They could be temperature, pressure, liquid level, or liquid flow rate in a chemical reaction process control problem. The list goes on and on. Now you realize that control is ubiquitous in almost all automated processes. Control is referred to as the hidden technology in engineering because it is so important to many devices, such as hard disk drives, and automated process but it is mainly out of sight. The principle underlying all control systems is feedback. As a human being, we have used the principle of feedback all the time in our daily lives. Let us use a very simple example to illustrate the feedback principle: the shower loop example. First, we have a desired shower temperature.

Then, we turn the knob, and water comes out. We use the hand to test the temperature. If it is too cold, we turn the knob towards the hot water side. If it is too hot, we turn the knob towards the cold water side until the water temperature is close to the preferred shower temperature. When the control system nomenclature is used, the desired temperature is called "reference input", "set point", "desired input", etc.; the water temperature is called the "output". The goal here is to let the output to be close to the "input". Our hand in this example serves as a "sensor" which is able to measure the output. The brain can compare the measured output and the desired input to know the difference between them. Then based on the error, our brain is able to make a decision. In a control system, the brain is called "controller". The hand and the knob in this case is the "actuator", which execute the command sent by the brain. The shower system is the one being controlled. It is called the "process". By putting all the components in a block diagram, we arrive at the generic control system diagram shown in Fig. 1.1.

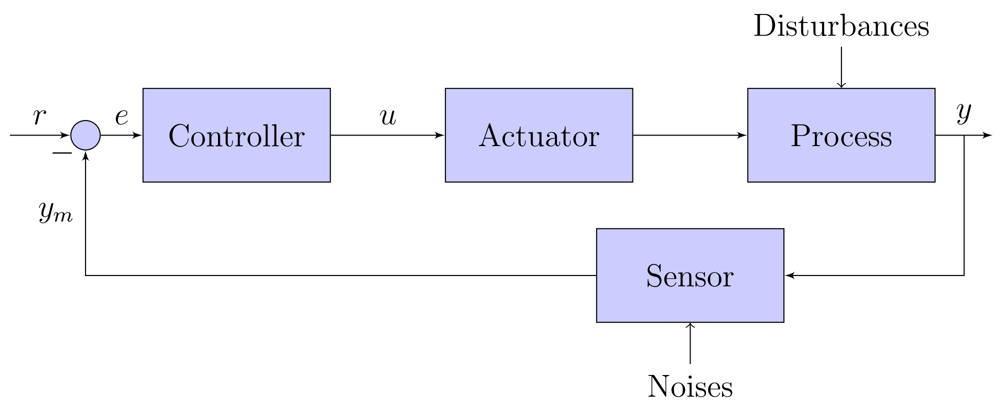


Figure 1.1: Control System Block Diagram

In a control system, there are four major signals

- Reference input (r): it is also called set-point.
- Output (y): it is the variable we want to control.
- Error (e): it is the difference between the reference input and the sensor output. Many control design is based on the error signal.
- Control signal (u): it is the signal computed based on the error and applied to the actuator.

and four major components:

- Process: it is the central component, the one being controlled. The process is usually subject to disturbances.
- Sensor: the role of the sensor is to measure of the output of the process. Sensors are required to measure the output with a high accuracy even though there are always sensor noises. It is required to be reliable and the sensor output has a linear relationship with the input. It should be sensitive to the input change.
- Controller: the controller is to compute a control signal based on the error. In traditional control, the controller is a circuit. In modern digital control, the controller is usually a micro-computer.
- Actuator: actuator is the one perform the action to control the process directly. The actuator needs to be powerful, speedy and reliable. It should be able to influence the process output in a large range and in a short period of time.

When we use the feedback principle to perform certain tasks in in the daily life, it is called "manual" control. When we teach the machine to use the feedback principle to perform certain tasks, it is called "automatic" control. No matter it is manual control or automatic control, it can be abstracted as the block-diagram shown in Fig. 1.1. We can use the room temperature control as an example. It is a case of automatic control of keeping the room temperature at a desired temperature for the comfort. In this example, the process is the house. Thermostat serves as both the sensor and the controller. The actuator is the valve, and the control signal is on and off of the heating and cooling systems as well as any ventilation fans that might be part of the system. The set point is the desired room temperature. Nowadays, most digital thermostats are programmable, and different step points can be set for different times of a day.

Further control system will incorporate the artificial intelligent, computer logic and sensor fusion into the feedback control to achieve intelligent control.

1.2 Open loop and Closed-loop Control

A control system can be open-loop or closed-loop. Even though the closed-loop control is the main stream for automatic control. Open-loop control has also been used in applications with a less requirement in accuracy. The open-loop control structure is shown in Fig. 1.2. In the open-loop control,

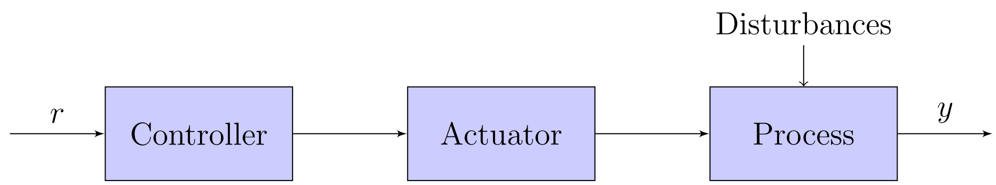


Figure 1.2: Open-loop Control System

the controller is calibrated based on many experiments to achieve a desired output for a given input signal. The open-loop control utilizes an actuating device to control the process directly without using feedback. Open-loop control is simple and inexpensive. Open-loop control is also used in the scenario when the output is hard to measure. But open-loop control is sensitive to disturbances and is not able to reject these disturbances. Home appliances usually use open-loop control, such as toasters, washing machine, microware, etc. The toaster cannot adjust its toast time according to the thickness of the toast. Anyone with burnt toast can attest that toaster operates in open-loop. In addition, there is no sensor to measure the color of the toast. There does not exist a sensor to measure the cleanness of clothes.

Closed-loop control uses a measurement of the output and feedback of the output signal to compare with the desired reference input. The control signal is computed based on the error between the two. Closed-loop control is less sensitive to noises, disturbances, and changes in the environment. The disadvantages are that closed-loop control has a complex structure and expensive when compared with open-loop control. Closed-loop control can be found in solar collectors, temperature control systems, speed control systems, etc.

1.3 Examples of Control Systems

Control systems have a long history, which can trace back to antiquity.

1.3.1 Flush Toilet

The flush toilet today still uses the principle of feedback to control the water level in the tank. Figure 1.3 shows how a flush toilet operates. The "float ball" rises as the tank fills with water. As it rises, the float rod attached to it presses against the inlet valve. When the tank is full, the rod is pressing against the inlet valve hard enough to turn the water off. This stops the tank from overflowing. This mechanism of the liquid-level control was invented in the ancient time.

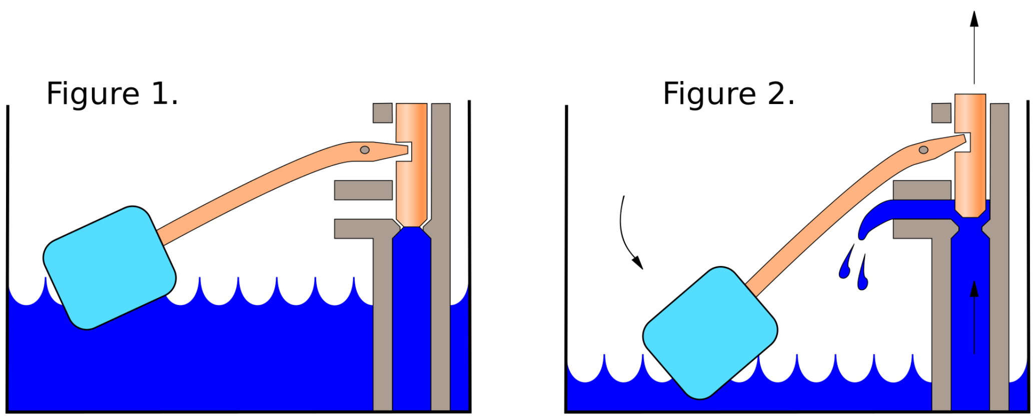


Figure 1.3: Flush toilet. Source: https://en.wikipedia.org/wiki/Flush_toilet

1.3.2 Unmanned Aerial Vehicles

Unmanned aerial vehicles become popular since 2010s. The quadcopters for personal entertainment found applications in tourism, light show, etc. Control systems are used in quadcopters to achieve various motions, such as tiling to a certain roll angle. Here the reference input is the desired roll angle. The output is the actual roll angle. We use a sensor to measure the actual roll angle. An electronic control system is used to generate a control signal based on the error. Actuator in this case is the power systems and rotors. The process is the quadcopter.

1.3.3 Autonomous vehicles

Control systems are expected to play a major role in self-driving cars, also known as autonomous vehicles. Some of the techniques have already been

Figure 1.4: Quadcopter

used in vehicles running on the road, such as lane keeping, adaptive cruise control.

Lane Keeping

Lane keeping provides automatic steering and braking to keep a vehicle in its travel lane. This feature relies on painted lane makings to operate. These include the markings between lanes and along the edges of the roads. This feature may help the driver from driving off the road and avoid collisions. Integrated video cameras track the road markings and detect any change in the distance to either side of the vehicle. Whether cameras, sensors or radar are used, all of these technologies can make our roads safer. After all, according to the German Federal Statistical Office, one third of accidents occurring outside built-up areas are the result of vehicles "veering off the road". Lane Keeping Assist can help eliminate this type of accident.

Adaptive Cruise Control

Adaptive cruise control is an advanced cruise control technique for road vehicles that automatically adjust the vehicle speed to maintain a safe distance from vehicles ahead (see Fig. 1.5). Control is based on sensor information

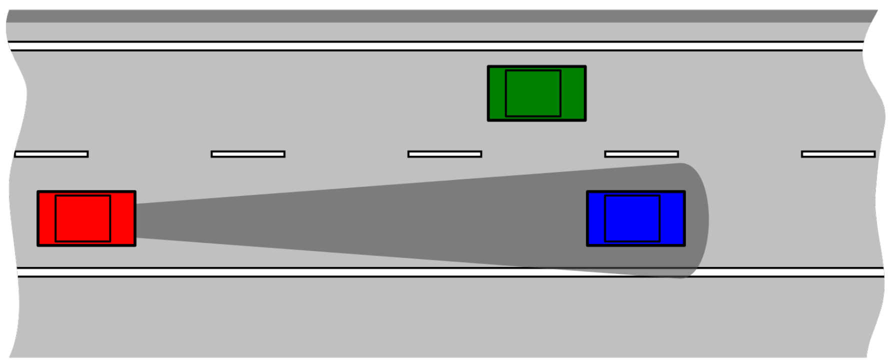


Figure 1.5: Adaptive Cruise Control. Source: https://en.wikipedia.org/wiki/Adaptive_cruise_control

from on-board sensors. Such systems may use a radar or laser sensor or a camera setup allowing the vehicle to brake when it detects the car is approaching another vehicle ahead, then accelerate when traffic allows it to. When combined with another driver assist feature such as lane centering, the vehicle is considered a Level 2 autonomous car.

1.4 Control System Design Process

The general steps to solve a real control problem:

- Choose hardware (sensors, and actuators).
- Develop mathematical models for all components (plant, sensor, actuator, etc.).

- Design controller based on the models.
- Simulate the designed controller using computer aid software. If the performance is unsatisfactory, identify the problem and re-design the control system.
- Test the control system on real plants. If the physical testing is unsatisfactory, redo all steps.

Control System Design is generally a trail-and-error process in which the above steps are used iteratively to determine the design parameters of an "acceptable" system. Acceptable performance is generally defined in terms of time and frequency domain criteria such as rise time, settling time, peak overshoot, gain and phase margin, and bandwidth.

Part I

Mathematics and Physics for Control Engineering

Chapter 2

Mathematics

2.1 Trigonometric functions

The linear combination of sine and cosine function is equivalent to a single sine or cosine function with a phase shift and scaled amplitude,

$$a \cos\theta + b \sin\theta = c\cos(\theta - \varphi_1) = c\sin(\theta + \varphi_2) \tag{2.1}$$

where c, φ_1 and φ_2 are defined as

$$\begin{aligned} c &= \sqrt{a^2 + b^2}, \\ \varphi_1 &= \arctan(b/a), \\ \varphi_2 &= \arctan(a/b). \end{aligned}$$

2.2 Complex Variables

A complex variable $z = a + bj$ is the sum of a real number and an imaginary number, where $a = \mathrm{Re}\{z\}$ is the real part of z, and $b = \mathrm{Im}\{z\}$ is the imaginary part of z. A complex variable can be written in the polar form as

$$z = r\exp(j\theta)$$

where $r = \sqrt{a^2 + b^2}$ and $\theta = \arctan(b/a)$. Note that $a = r\cos(\theta)$ and $b = r\sin(\theta)$. Here r is the magnitude of z, denoted as $|z|$, and the angle θ can be denoted by the form $\underline{/z}$. The conjugate of the complex variable

$z = a + bj$ is denoted by z^* and is defined as $z^* = a - bj$. The multiplication and division of two complex numbers $z_1 = r_1\underline{/\theta_1}$ and $z_2 = r_2\underline{/\theta_2}$ are

$$z_1 z_2 = (r_1\underline{/\theta_1})(r_2\underline{/\theta_2}) = r_1 r_2\underline{/\theta_1 + \theta_2}$$

and

$$\frac{z_1}{z_2} = \frac{r_1\underline{/\theta_1}}{r_2\underline{/\theta_2}} = \frac{r_1}{r_2}\underline{/\theta_1 - \theta_2}.$$

2.3 Derivative

Here are the rules for the derivatives of the most common basic functions.

Derivative of powers

$$\frac{d}{dx} x^a = a x^{a-1},$$

where a is an integer and x is positive.

Exponential functions

$$\frac{d}{dx}\exp(x) = \exp(x).$$

Trigonometric functions

$$\frac{d}{dx}\sin(x) = \cos(x).$$

$$\frac{d}{dx}\cos(x) = -\sin(x).$$

Rules for combined functions

- Constant rule: if $f(x)$ is constant, then

$$f^{(1)}(x) = 0.$$

- Sum rule: if $f(x)$ and $g(x)$ are functions, α and β are constants, then

$$(\alpha f + \beta g)^{(1)} = \alpha f^{(1)} + \beta g^{(1)}.$$

- Product rule: if $f(x)$ and $g(x)$ are functions, then
$$(fg)^{(1)} = f^{(1)}g + fg^{(1)}.$$
- Quotient rule: if $f(x)$ and $g(x)$ are functions, then
$$\left(\frac{f}{g}\right)^{(1)} = \frac{f^{(1)}g - fg^{(1)}}{g^2}.$$
- Chain rule for composite functions: if $f(x) = h\left(g(x)\right)$, then
$$f^{(1)}(x) = \frac{\partial h\left(g(x)\right)}{\partial g(x)} \cdot g^{(1)}(x).$$

Example 1 *Calculate the derivative of the function*
$$f(x) = x^4 + \sin\left(x^2\right) + 7.$$

2.4 Laplace Transforms

The Laplace transform is a linear operator of a function $f(t)$ with a real argument t ($t \geq 0$) that transforms $f(t)$ to a function $F(s)$ with a complex argument s, given by the integral
$$F(s) \triangleq \int_0^\infty f(t)e^{-st}dt,$$
where $s = \sigma + \omega j$ is a complex frequency parameter with real numbers σ and ω. An alternate notation for the Laplace transform is $\mathscr{L}\{f\}$.

Example 2 *Find the Laplace transform of the impulse function $\delta(t)$.*

Example 3 *Find the Laplace transform of the step $f(t) = a$ and ramp $f(t) = bt$ functions.*

2.4.1 Properties of Laplace Transforms

Superposition

The Laplace transform is linear, which means that the principle of superposition applies:
$$\mathscr{L}\{\alpha_1 f_1(t) + \alpha_2 f_2(t)\} = \alpha_1 F_1(s) + \alpha_2 F_2(s).$$

Differentiation

The transform of the derivative of a signal is related to its Laplace transform and its initial condition as follows:

$$\mathscr{L}\left\{f^{(1)}\right\} = sF(s) - f(0) \tag{2.2}$$

where $f^{(1)}$ is the first derivative of f. We denote the derivative of $f^{(1)}$ by $f^{(2)}$ and call $f^{(2)}$ the second derivative of f. Continuing in this manner, we obtain functions $f^{(n)}$ is called the nth derivative of f. Another application of (2.2) leads to

$$\mathscr{L}\{f^{(2)}\} = s^2F(s) - sf(0) - f^{(1)}(0).$$

Repeated application of (2.2) leads to

$$\mathscr{L}\{f^{(n)}\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f^{(1)}(0) - \cdots - s^0f^{(n-1)}(0).$$

Integration

The Laplace transform of the integral of a signal $f(t)$ is to simply divide the Laplace transform of the function by s, that is,

$$\mathscr{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}.$$

Convolution

The convolution of two functions f and g is written $f(t) * g(t)$, and its definition is

$$f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)d\tau.$$

Convolution in the time domain corresponds to multiplication in the frequency domain, that is,

$$\mathscr{L}\{f_1(t) * f_2(t)\} = F_1(s)F_2(s),$$

where $\mathscr{L}\{f_1(t)\} = F_1(s)$ and $\mathscr{L}\{f_2(t)\} = F_2(s)$.

Time Shifting

Let $f(t-\tau)$ be a function which is delayed by τ seconds of $f(t)$. Then,

$$\mathscr{L}\{f(t-\tau)\} = \exp(-\tau s)F(s).$$

2.5 Cramer's Rule

Cramer's rule is an explicit formula for the solution of a system of linear equations with as many equations as unknowns, valid whenever the system has a unique solution. Consider a system of n linear equations for n unknowns, represented in matrix multiplication form as follows:

$$Ax = b,$$

where the $n \times n$ matrix A has a nonzero determinant, and the vector $x = [x_1, \ldots, x_n]^T$ is the column vector of the variables. Then Cramer's rule states that the system has a unique solution, whose individual values for the unknowns are given by

$$x_i = \frac{\det(A_i)}{\det(A)},$$

where A_i is the matrix formed by replacing the i-th column of A by the column vector b.

Chapter 3

Physics

3.1 Newton's laws of motion

3.1.1 Inertial Reference Frame

In classical physics, an inertial frame of reference is a frame of reference that is not undergoing acceleration. The motion of a body can only be described relative to something else - other bodies, observers, or a set of spacetime coordinates. If the coordinates are chosen badly, the laws of motion may be more complex than necessary. Take the example of a person driving a car. If we take the road as the inertial reference frame, the car is moving forward, and trees along the road side is not moving. However, if we take the car as the inertial reference frame, then the car is not moving, but the trees are moving behind.

3.1.2 Spring and Damper

Spring

Hooke's law of physics that states that the force F_s needed to extend or compress a spring by some distance x scales linearly with respect to that distance, that is,

$$F_s = kx$$

where k is a constant factor characteristic of the spring, and x is the distance of the free end of the spring from its "relaxed" position.

Damper

A damper is a mechanical device which resists motion via viscous friction. The resulting force is proportional to the velocity, but acts in the opposite directions, slowing the motion and absorbing energy. It is commonly used in conjunction with a spring. The viscous friction is

$$F_v = bv,$$

where b is the viscous friction constant, and v is the velocity. The symbol for a damper is shown in Fig. 3.1.

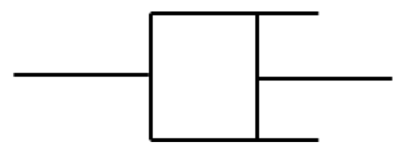


Figure 3.1: Damper

3.1.3 Translational and Rotational Motions

Newton's laws of motion are three physical laws that, together, laid the foundation for classical mechanics.

First law

In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon a force.

Second law

In an inertial frame of reference, the sum of all forces f on an object is equal to the mass m of that object multiplied by the acceleration a of the object, that is,

$$f = ma.$$

Since

$$a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$$

and

$$v = \frac{dx}{dt},$$

the following relationships hold by applying the Laplace transform $F(s) = mA(s)$, $F(s) = msV(s)$, and $F(s) = ms^2X(s)$ by assuming zero initial conditions.

In an inertial frame of reference, the sum of all torques τ on an one-dimensional rotational system is equal to the moment of inertia I multiplied by the angular acceleration α, that is,

$$\tau = I\alpha.$$

Here the torque is the tuning effect of a force

$$\tau = fr,$$

where f is the force applied perpendicular to the rotational system, and r is the distance from the place where the force is applied to the rotational center, and the moment of inertial I is a measure of the object's resistance to changes to its rotation, where $I = mr^2$. Similar to the translational motion, we have the relationships

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

and

$$\omega = \frac{d\theta}{dt},$$

where θ is the angle, ω is the angular velocity. By applying the Laplace transform, the above relationships can be written as

$$\alpha(s) = s\Omega(s) = s^2\Theta(s)$$

in the complex domain by assuming zero initial conditions.

Third law

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

3.2 Circuits

3.2.1 Electric circuits

Below we show some elements of electric circuits: Resistor, Capacitor and Inductor. The relationships between the voltage v across and current i through the elements are shown in Table 3.1. It is more convenient to use the impedance rather than the differential equations. In Table 3.1, C is the capacitance, L is the inductance, R is the resistance, and s is a complex variable.

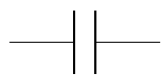
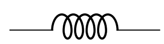
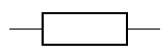
	Symbol	Equation	Impedance
Capacitor		$i = C\frac{dv}{dt}$	$\frac{V}{I} = \frac{1}{Cs}$
Inductor		$v = L\frac{di}{dt}$	$\frac{V}{I} = Ls$
Resistor		$v = Ri$	$\frac{V}{I} = R$

Table 3.1: Basic Electric Elements

3.2.2 Kirchhoff's circuit laws

Kirchhoff's circuit laws include Kirchhoff current law and Kirchhoff voltage law.

Kirchhoff's current law

This law states that, for any node in an electrical circuit, the sum of currents flow into that node is equal to the sum of currents flowing out of that node; or equivalently

The algebraic sum of currents leaving a node equals to the algebraic sum of currents entering that node.

Recalling that current is a signed quantity reflecting direction towards or away from a node, this principle can be succinctly stated

$$\sum_{k=1}^{n} I_k = 0,$$

where n is the total number of branches with currents flowing towards or away from the node. For the circuit shown in Fig. 3.2, Kirchhoff's current law suggests that

$$i_2 + i_3 = i_1 + i_4.$$

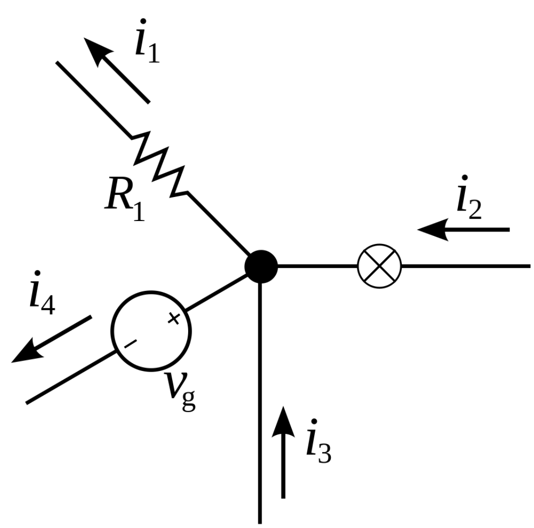


Figure 3.2: Kirchhoff's current law. Source: https://en.wikipedia.org/wiki/Kirchhoff's_circuit_laws

Kirchhoff's voltage law

This law states

The algebraic sum of all voltages taken around a closed loop in a circuit is zero.

Similarly to Kirchhoff's current law, the voltage law can be stated as

$$\sum_{k=1}^{n} V_k = 0,$$

where n is the total number of voltages measured. For the circuit shown in Fig. 3.3, Kirchhoff voltage law suggests that

$$v_1 + v_2 + v_3 - v_4 = 0,$$

if the voltage is positive if the current flows from a high voltage to a low voltage, and it is negative otherwise.

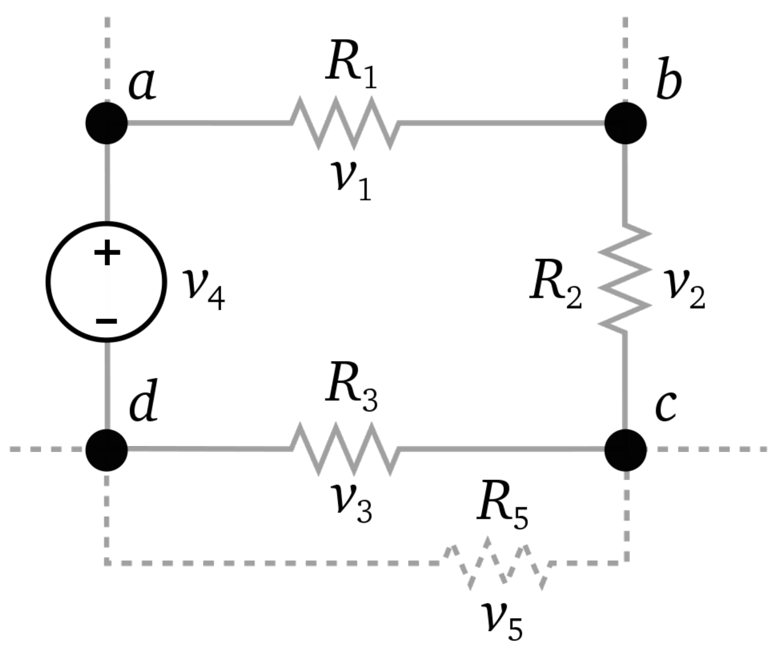


Figure 3.3: Kirchhoff's voltage law. Source: https://en.wikipedia.org/wiki/Kirchhoff's_circuit_laws

3.2.3 Operational Amplifier

In control systems, one additional circuit element is usually present – the operational amplifier. Operational amplifier also known as op amp is normally used in sensor circuit to amplify weak signal, and is also used in compensation circuit. Operational amplifier is an electronic amplifier used as a building block to implement transfer function. The op amp consists of two inputs: noninverting input (V_+) and inverting input (V_-). The operational amplifier is designed and constructed such that the input impedance is very high, resulting in i_+ and i_- being very small.

Open-loop Operational Amplifier

A simplified circuit of an open-loop operational amplifier is shown in Fig. 3.4. It has a differential input. The output voltage of the op amp

$$V_{out} = A_{OL}(V_+ - V_-),$$

where A_{OL} is the open-loop gain of the amplifier. Additionally, the amplifier gain is very large. However, the magnitude of A_{OL} is not well controlled

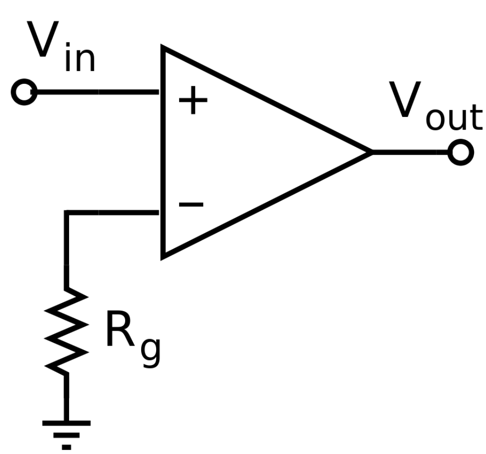


Figure 3.4: Open-loop Operational Amplifier. Source: https://en.wikipedia.org/wiki/Operational_amplifier

by the manufacturing process, and so it is impractical to use an open-loop amplifier as a stand-alone differential amplifier.

Closed-loop Operational Amplifier

In the non-inverting amplifier shown in Fig. 3.5, the negative feedback is achieved via the voltage divider R_f and R_g. Equilibrium is established when

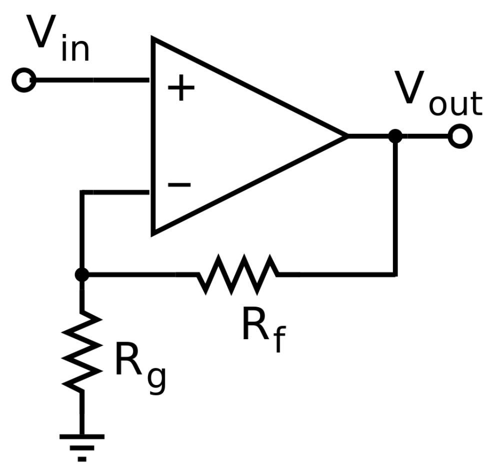


Figure 3.5: Closed-loop Operational Amplifier. Source https://en.wikipedia.org/wiki/Operational_amplifier

the positive and negative terminals of the op amp have the same voltage, that is, $V_+ = V_- = V_{in}$. Based on the voltage division, we have

$$\frac{V_{in}}{R_g} = \frac{V_{out}}{R_g + R_f}.$$

Therefore,

$$V_{out} = \left(1 + \frac{R_f}{R_g}\right) V_{in}.$$

Ideal Operational Amplifier Characteristics

We assume that all operational amplifiers used in control circuits are ideal. An ideal op amp is usually considered to have the following characteristics:

- Infinite Input Impedance: Another quirk of the ideal Operational Amplifier is the infinite input impedance. The operational amplifier has infinite input impedance which makes it useful in the signal conditioning applications where it is required to input less current to avoid the attenuation already weak signal, in instrumentation and many more. The infinite input impedance of the Operational Amplifier is very desirable as it result in zero input current.

- Zero Output Impedance: Another important feature of the ideal Operational Amplifier is the zero output impedance which makes it very desirable in the signal amplifier. Because of the zero output impedance of the Operational Amplifier it act as the Ideal Voltage Source so with any amount of current flowing through the Op-AMP the signal strength will not be disturbed. As the zero output impedance will result zero voltage drop at the output terminals of the Operational Amplifier so the signal will not get affected irrespective of the amount of current that flows through the Operational Amplifier.

Infinite input impedance means that the amplifier does not draw current (load) the previous stage. All the previous stage output voltage apply to the amplifier input without current, so the previous stage does not consume additional power from power supply (higher efficiency), and the amplifier does not make input signal distorted. In the same way, the amplifier with zero output impedance will load all output voltage to the next stage and the output signal will not be distorted. The output voltage is not affected by the load connected to the output terminal.

The formulas for ideal operational amplifier are

$$i_+ = i_- = 0, \tag{3.1}$$

$$v_+ = v_-. \tag{3.2}$$

Note that the current flowing into the op amp is zero due to the infinite impedance. However, the current out of the op amp is NOT zero.

3.3 Mechatronics

3.3.1 Law of Motors

Consider a conductor of length l m is arranged in a magnetic field of flux density B teslas carrying a current I amp. Then there is a force on the conductor. The size of this force is

$$F = Blisin(\theta),$$

where θ is the angle made by the conductor with the magnetic field. This equation is the basis of conversion of electric energy to mechanical work and is called the law of motors. The direction of the force can be found by application of the right hand rule as follows: thumb follows the direction of current flow, the middle figure points to the polarities from N to S. The direction where the palm faces is the direction of the force.

3.3.2 Law of Generator

If a straight conducting wire is placed in a plane perpendicular to a uniform magnetic field, and is moving in a direction perpendicular to the field, then each charge q in the wire experiences a magnetic force of magnitude $F = qvB$. The negatively charged electrons will accelerate in response to this force. Since they cannot leave the wire, negative charge will accumulate on one end of the wire, while positive charge will be left behind on the other end. The separated charges produce an electric field, which exerts a force on the other charges in the wire. This electric force opposes the magnetic force. If we place the wire on a conducting rail, a current will start to flow in the circuit formed by the rail and the wire. The voltage across the wire is

$$e = Blv,$$

where B is the magnetic filed density, l is the length of conductor, and v is the velocity. The direction of the current follows the right hand rule: thumb follows the direction of motion, the middle points to the direction of the magnetic field from N to S, the palm faces the direction of the current.

Part II

Analysis and Design of Control Systems

Chapter 4

Modeling of Dynamic Systems

The learning objectives of this chapter:

- Recognize that transfer functions can describe the dynamic behavior of physical systems;
- Understand the application of Laplace transforms and their role in obtaining transfer functions;
- Model the basic mechanical systems;
- Model the basic electrical systems;
- Model the basic mechatronic systems;
- Understand the important role of modeling in the control system design process.

A mathematical modeling is a description of a system using mathematical concepts and language. To model the dynamics of a control system, we can use differential equations. In traditional control theory, it is more convenient using a transfer function to represent the input-output relationship in a frequency domain. The transfer function $H(s)$ of an LTI system is defined as the ratio of the Laplacian transform of the output of the system to its input assuming all zero initial conditions.

A control system consists of four major components: process, sensor, controller and actuator. The process combined with the actuator are called the "plant". Here we will discuss how to model each component using the underlying physics.

4.1 Modeling the Process

4.1.1 Autonomous Vehicle Cruise Control

Here we use a simplified model for the vehicle dynamics. Let us use x to denote the distance from the reference 0 distance point. The inertia reference frame is defined as the positive direction to the right. To find the transfer function, we first draw the free body diagram. A free body diagram is a graphical illustration used to visualize the applied forces, movements, and resulting reactions on a body in a given condition. The free body diagram of the autonomous vehicle cruise control is shown in Fig. 4.1. Here u is the engine torque. It is the input. According to Newton's second law, we have

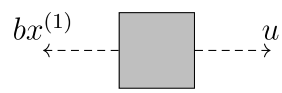


Figure 4.1: Free Body Diagram

$$u - bx^{(1)} = mx^{(2)},$$

where b is constant which is related to the air resistance. Here x is the output and u is the input. Applying the Laplace transform, we have

$$U(s) - bsX(s) = ms^2X(s)$$

if we assume all initial conditions are zeros. The transfer function is

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + bs}.$$

To obtain the transfer function from the engine torque to the velocity, we can use the relationship $sX(s) = V(s)$. Then

$$\frac{V(s)}{U(s)} = \frac{sX(s)}{U(s)} = \frac{1}{ms + b}.$$

4.1.2 Quadcopter Drones

Figure 4.2 shows a quadcopter with four rotors. Two of them rotates in the clockwise direction and the other two rotates in the counterclockwise direction. The rotor blades are arranged so that the rotating rotor blades generate lifting forces. The quadcopter drone hovers in the air when the lifting force is balanced with the gravity force applied to the quadcopter. By changing the rotational speed of the four rotors, various translational and rotational motions can be made by quadcopters. The torques to the fours

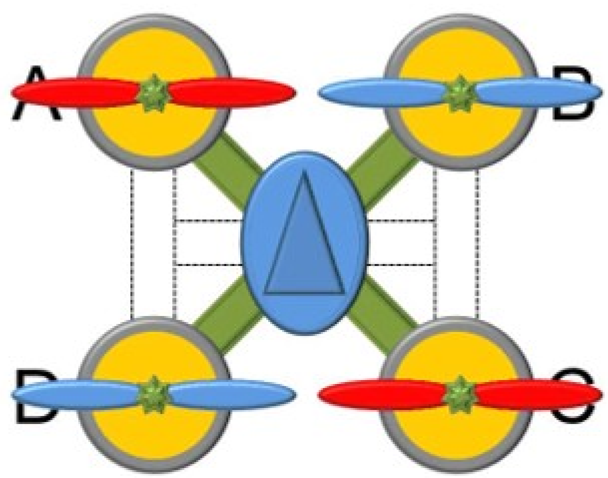


Figure 4.2: Quadcopter. Source: https://en.wikipedia.org/wiki/Quadcopter

rotor are defined as T_A, T_B, T_C, and T_D. For the rotational motion, the quadcopter can make the roll, pitch, and yaw motions. The definitions are given below:

- Roll: rotate around the front-to-back axis;
- Pitch: rotate around the side-to-side axis;
- Yaw: rotate around the vertical axis.

To produce the pitch motion, we can increase the rotational speeds of rotors C and D, and decrease the rotational speeds of rotors A and B. To produce the roll motion, we can increase the rotational speeds of A and B, and decrease the rotational speeds of B and C. To produce the yaw motion, we can increase the rotational speeds of A and C, and decrease the rotational speeds of B and

D. To make the drone hovering in the air, the increase amount of the speed should be the same with the decrease amount of the speed. Let us denote the control torque for positive θ motion as T_θ. Then the transfer function from the torque to the pitch angle θ is

$$\frac{\Theta(s)}{T_\theta(s)} = \frac{1}{I_y}\frac{1}{s^2}.$$

This model is called a double integrator model because it contains the product of two $1/s$, which corresponds to the Laplace transform of an integration.

4.2 Modeling controllers

In the last section, newton's law is used to model the process, which is usually a mechanical system. This section devotes to the modeling of electric circuits. Electric circuits are used frequently in control systems, especially in controllers. This is because electric signal is easy to be processed and manipulated. Although the controller is more and more implemented with digital logic by microprocessors and computers, analog circuits are still used to perform certain functions. There are two seasons: one is that analog circuit is faster; and it is cheaper than a digital implementation for a simple controller. A sensor is a device whose purpose is to convert the process output into an electrical signal so that it can be easily manipulated by controllers. Furthermore, analog circuits must be used in power amplifier for mechatronic control and anti-alias pre-filters for digital control.

Electric circuits consist of the intersections of the power sources such as voltage and current and other electronic components, such as resistor, capacitor, inductor. A very important building block in control system design is the operational amplifier, which is short for Op-amp. It is self a negative feedback control system.

A PID controller circuit is shown in Fig. 4.3. To find the transfer function of the PID controller, we can find the impedance first. The impedance of the parallel connection of R_1 and C_1 is

$$Z_1 = \frac{R_1}{R_1C_1s + 1}.$$

The impedance of the series connection of R_2 and C_2 is

$$Z_2 = \frac{1 + C_2R_2s}{C_2s}.$$

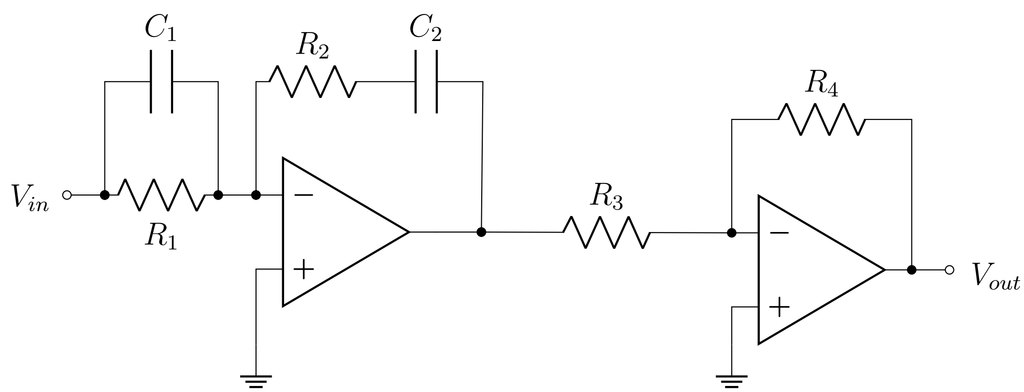


Figure 4.3: PID Controller

Let us denote the output voltage of the first operation amplifier is V. Then the transfer function from V_{in} to V is

$$\frac{V(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{1 + C_2R_2s}{C_2s}\frac{R_1C_1s + 1}{R_1}.$$

The transfer function from V to V_{out} is

$$\frac{V_{out}(s)}{V(s)} = -\frac{R_4}{R_3}.$$

Therefore, the PID controller can be written as

$$\frac{V_{out}}{V_{in}} = k_P + \frac{k_I}{s} + k_D s,$$

where

$$k_P = \frac{R_4}{R_2}\frac{C_1}{C_2} + \frac{R_2R_4}{R_1R_3} \tag{4.1}$$

$$k_I = \frac{R_4}{R_1C_2R_3} \tag{4.2}$$

$$k_D = \frac{C_1R_2R_4}{R_3} \tag{4.3}$$

By tuning the electric elements of resistor and capacitor, the parameters of the PID controller are adjusted.

4.3 Modeling sensors and actuators

Electric current and magnetic fields interact in two ways that are particularly important to an understanding of the operation of most electromechanical actuators and sensors.

4.3.1 Sensors

Position Sensor

A position sensor is a sensor that facilitates measurement of mechanical position. A position sensor may indicate absolute position (location) or relative position (displacement), in terms of linear travel, rotational angle, or three-dimensional space. Common types of position sensors include potentiometer and encoder. A potentiometer is a three-terminal resistor with a sliding or

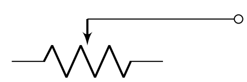


Figure 4.4: Position Sensor

rotating contact that forms an adjustable voltage divider, as illustrated in Fig. 4.4. Let us assume the resistance of the resistor is R, and the right part of the voltage divider is R_1. Then the voltage across R_1 is

$$V_s(t) = \frac{R_1}{R} V,$$

where V_s is the sensor output, V is the applied DC voltage, and R_1 is proportional to the displacement. Therefore, the mechanical position is translated into the voltage.

Similar to optical position sensors, *ultrasonic position sensors* emit a high-frequency sound wave generated typically from a piezoelectric crystal transducer. The ultrasonic waves generated from the transducer are reflected from the object being measured, or target, back to the transducer where an output signal is generated. Ultrasonic sensors can function to perform as proximity sensors, where they report on an object being within a specified range of the sensor, or as a position sensor which provides ranging information. The advantages of ultrasonic position sensors are that they can work with target objects of different materials and surface characteristics, and can detect small objects over larger distancer than other types of position sensors.

A passive infrared sensor (PIR sensor) is an electronic sensor that measures infrared (IR) light radiating from objects in its field of view. They are most often used in PIR-based motion detectors. PIR sensors are commonly used in security alarms and automatic lighting applications.

Throttle position sensors provide feedback to the fuel injection system of an automobile. Motion sensors detect the movement of an object and can be used to trigger action (such as illuminating a floodlight or activating a security camera). Proximity sensors as well can detect that an object has come within range of the sensor. Both sensors, therefore, might be considered as a specialized form of position sensors.

Velocity Sensor

A special DC generator, called tachometer, can be used to measure velocity. A tachometer is an instrument measuring the rotation speed of a shaft or disk, as in a motor or other machine. The word tach comes from Greek which means speed. Tachometers or revolution counters on cars, aircraft, and other vehicles show the rate of rotation of the engine's crankshaft, and typically have markings indicating a safe range of rotation speeds. According to the law of generator, the output of the sensor

$$e = K_e \omega.$$

The tachometer output voltage is directly proportional to its shaft velocity.

Acceleration Sensor

An accelerometer is an electromechanical device used to measure acceleration forces. Such forces may be static, like the continuous force of gravity or, as is the case with many mobile devices, dynamic to sense movement or vibrations. Acceleration is the measurement of the change in velocity.

4.3.2 Actuator

Motor is one of the most common actuators. An electric motor is an electrical machine that converts electrical energy into mechanical energy. Most electric motors operate through the interaction between the motor's magnetic field and electric current in a wire winding to generate force in the form of torque applied on the motor's shaft. Electric motors are the most common actuators in a control system. Electric motors can be found in most electronic devices, such as computer hard-disk drive control, electric vehicles, quadcopters, washing machines, etc. Electric motors work under the principles of law of motors and law of generators. The stator is the part which does not move. Two magnet on the side. The armature in the middle is also called a rotor, which is the part that spins. The axle that sticks out of the motor is a shaft. The input of the motor is usually a voltage source. The circuit connects the commutator with two brushes on the side. The current flows through the brush, commutator, and armature loop and back through the other side. The commutator will spin along the armature. The brushes slide along as the commutator spin, but they always maintain contact with the commutator. According to the law of motor, there is a force on the armature when a current carry armature is placed in a magnetic filed at the right angle.

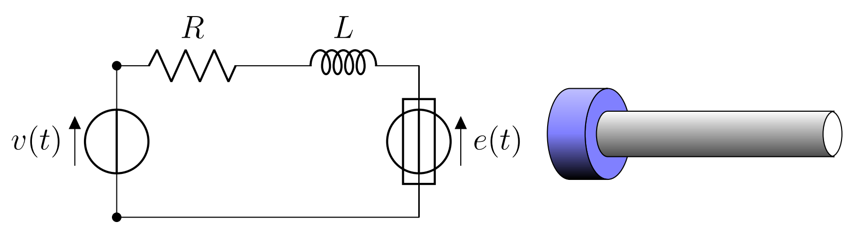


Figure 4.5: Electric Motors

An equivalent electric circuit of a DC motor is shown in Fig. 4.5. According to the motor equations, the torque T on the rotor and the back electromotive force voltage are given by

$$T = K_t i_a \tag{4.4}$$
$$e = K_e \theta_m^{(1)} \tag{4.5}$$

where K_t is the torque constant, and K_e is the electric constant. Based on Newton's laws,

$$K_t i_a - b\theta_m^{(1)} = J_m \theta_m^{(2)}.$$

Based on KCL law,

$$L i_a^{(1)} + R i_a = v - K_e \theta_m^{(1)}.$$

By applying Laplace transform to both equations, we have

$$J_m s^2 \Theta_m + bs\Theta_m = K_t I_a(s) \tag{4.6}$$

$$K_e s\Theta_m + LsI_a(s) + RI_a(s) = V(s) \tag{4.7}$$

The voltage is the input, and the rotational speed is the output. The transfer function from the voltage $V(s)$ to the rotational speed $\Omega(s)$ is

$$\frac{\Omega(s)}{V(s)} = \frac{s\Theta(s)}{V(s)} = \frac{K_t}{J_m L s^2 + (RJ_m + bL)s + K_t K_e}.$$

Chapter 5

Analysis of Control Systems

The learning outcomes in this chapter are shown as follows:

- Be ware of block diagrams and their role in analyzing control systems
- Understand the concept of stability of dynamic systems
- Be aware of the impact of zeros and poles on the system response.
- Recognize the direct relationship between the pole locations of second-order systems and the transient response.

5.1 Block Diagrams

In Chapter 4, the transfer function is obtained by first applying the Laplace transform to differential equations, and then algebraically solving the set equations. The algebraic equations after the Laplace transform can be arranged in a way that two blocks do not interact unless the output of a block is the input of another block. Each block represents the relationship between its input and output. The relationships between blocks are indicated by lines and arrows. When all components in a component block diagram are replaced by its corresponding transfer function, it becomes a block diagram. By viewing the transfer function inside each block as a number, the overall transfer function can be obtained by graphical simplification. It provides a different way to solve the algebraic equation.

5.1.1 Terminologies

Path: a sequence of connected blocks, the route passing from one node to another in the direction of signal flow of the blocks without including any block more than once.
Forward Path: a path from the input to output such that no node is included more than once.
Loop: Any closed path that returns to its starting node without passing through any node more than once.
Path Gain: The path gain is the product of transfer functions making up the path.
Loop Gain: the path gain associated with a loop - that is, the product of gains in a loop.

5.1.2 Elementary Block Diagram

Series Connection

Two subsystems G_1 and G_2 are connected in series as shown in Fig. 5.1, that is, the output of G_1 is the input of G_2. The transfer function for the overall system is

$$\frac{Y(s)}{R(s)} = G_1 G_2.$$

The overall transfer function is the path gain from input $R(s)$ to output $Y(s)$.

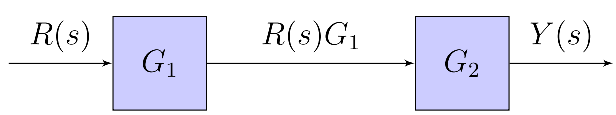


Figure 5.1: Series connection

To show this, the algebraic approach can be used. The output of G_1 is $R(s)G_1$, which is the input of G_2. Then, $Y(s) = R(s)G_1G_2$.

Parallel Connection

Two subsystems G_1 and G_2 are connected in parallel as shown in Fig. 5.2, that is, the subsystems G_1 and G_2 have the same input $R(s)$ and the sum of their outputs is the output of the overall system. The transfer function for the overall system is

$$\frac{Y(s)}{R(s)} = G_1 + G_2.$$

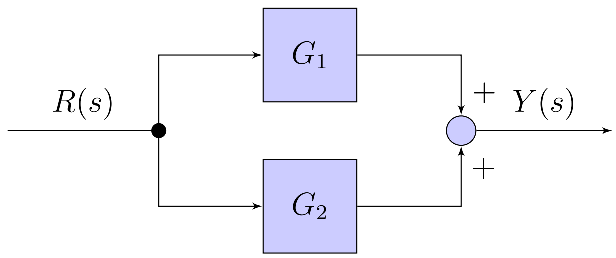


Figure 5.2: Parallel connection

The outputs of G_1 and G_2 are $R(s)G_1$ and $R(s)G_2$, respectively. The output $Y(s) = G_1R(s) + G_2R(s) = (G_1 + G_2)R(s)$.

Feedback Connection

Two subsystems are connected in a feedback arrangement as shown in Fig. 5.3. The output $Y(s)$ is

$$Y(s) = (R(s) - G_2Y(s))\, G_1.$$

By arranging the terms, we have

$$Y(s)(1 + G_2G_1) = R(s)G_1.$$

Therefore, the transfer function

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1G_2}.$$

A general rule for the feedback connection is that the transfer function is a ratio of the forward path gain and 1 minus the loop gain. Here the forward path gain is G_1 and the loop gain is $-G_1G_2$. Note that when the loop gain is calculated, all the signs along the loop must be included.

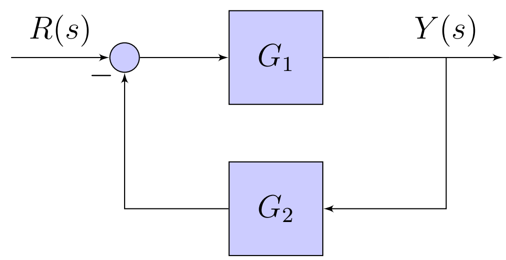


Figure 5.3: Block Diagram

Block Diagram Algebra

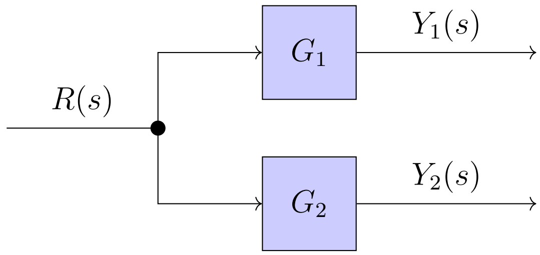
In the block diagram algebra, the pick-off point can be considered as parentheses, the parallel connection of two blocks is like a summation. We will discuss three algebraic operation of block diagrams.

- Moving a block across a pick-off point: The block diagram shown in Fig. 5.4(a) is equivalent to the block diagram shown in Fig. 5.4(b). From both block diagrams, we have $Y_1(s) = G_1 R(s)$ and $Y_2(s) = G_2 R(s)$.

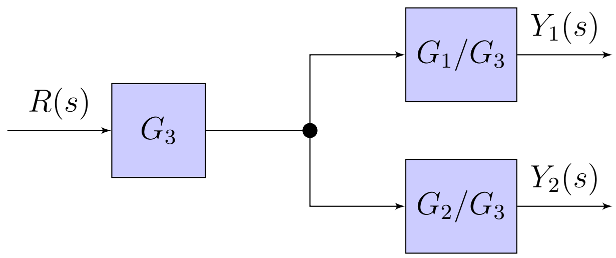
- Moving a block across a summer: The block diagram shown in Fig. 5.5(a) is equivalent to the block diagram shown in Fig. 5.5(b). From both block diagrams, we have $Y(s) = G_1(U_1(s) + G_2 U_2(s))$ and $Y(s) = G_2 G_1(U_1(s)/G_2 + U_2(s))$. When a block is moved from the input side of a summer to the output side of a summer, all input paths are divided by the moving block, and all output paths are multiplied by the moving block.

Block-diagram reduction is a way to change the block-diagram to different forms by moving blocks across pick-off points and summers so that it can be reduced to a simpler form to represent the input-output relationship.

Tips: In general, the block diagram reduction process is easier when pick-off points or summing points are combined into one by moving the pick-off points or summing points. There are no general rules of moving the summing point and the pick-off point.



(a)



(b)

Figure 5.4: Moving a pick-off point

5.2 Transfer Functions

5.2.1 Impulse Response

Two important properties of an LTI system are:

- The system satisfies the principle of superposition.
- The input signal is delayed, and the response of the output is the same except that it is delayed by the same amount of time.

If $y_1(t)$ and $y_2(t)$ are the outputs of an LTI system with the inputs $u_1(t)$ and $u_2(t)$, respectively. What is the output of the LTI system with the input $\alpha_1 u_1(t-\tau_1)+\alpha_2 u_2(t-\tau_2)$? The output is $\alpha_1 y_1(t-\tau_1)+\alpha_2 y_2(t-\tau_2)$.

Is there a "standard signal" so that if we know the response of the standard signal, then we know the response of every signal based on the two properties of the LTI system by scaling and shifting the response of the "standard signal"? The answer is yes, and such "standard signal" is the

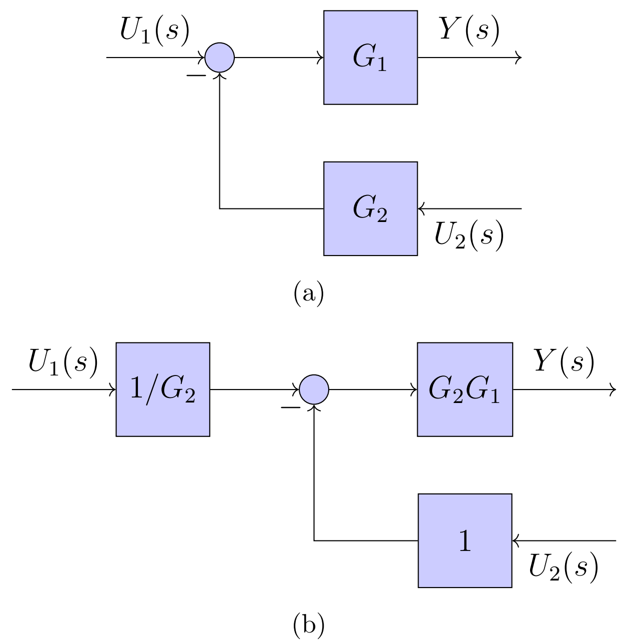


Figure 5.5: Moving a summer

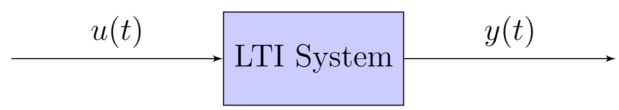


Figure 5.6: LTI System

impulse signal. Mathematically, the unit impulse signal is defined as

$$\delta(t) = 0 \text{ for } t \neq 0, \text{ and } \int_0^{+\infty} \delta(t)dt = 1. \tag{5.1}$$

Any input signal $u(t)$ can be written as

$$u(t) = \int_0^t u(\tau)\delta(t-\tau)d\tau$$

based on the sifting property or the sampling property of the delta function. The input signal is expressed as the integration (infinitely many sum) of impulses signals delayed by τ and scaled by $u(\tau)d\tau$ for $0 \leq \tau < +\infty$. The output $h(t)$ is called *impulse response* when the input signal is $\delta(t)$. Then according to the superposition principle and time-invariant property, the output of a system is

$$y(t) = \int_0^t u(\tau)h(t-\tau)d\tau \tag{5.2}$$

when the input is $u(t)$.

The output signal $y(t)$ of an LTI system with an input signal $u(t)$ can be described by the convolution integral of the input $u(t)$ with the unit impulse response of the system $h(t)$.

With the assumption of zero initial conditions of the system, (5.2) becomes

$$Y(s) = H(s)U(s)$$

when the Laplace transform is applied to both sides of (5.2), and

$$H(s) = \frac{Y(s)}{U(s)}.$$

The transfer function $H(s)$ of an LTI system is the ratio of the Laplacian transform of the output of the system to its input assuming all zero initial conditions.

The Laplace transform of the delta function is 1. Therefore, the Laplace transform of the impulse response of an LTI system is the transfer function, that is,

$$H(s) = Y(s).$$

Thus, another definition of the transfer function of an LTI system is given as

The transfer function $H(s)$ is the Laplace transform of the unit impulse response $h(t)$ of an LTI system assuming all zero initial conditions.

5.2.2 General Time Response

The steps to calculate the general time response of a system.

1. Determine of the model of the system in the form of transfer functions;
2. Determine the Laplace transform of the input signal $U(s)$;
3. Calculate the Laplace transform of the output signal $Y(s) = H(s)U(s)$;
4. Find the inverse Laplace transform of the output signal $Y(s)$, that is, the time response $y(t)$.

5.2.3 Partial Fraction Expansion

If the Laplace transform of the output signal is a rational function, that is, the ratio of two polynomials in s, then a simple method called "partial-fraction expansion" can be used to calculate the time response easily. Consider the general rational form of $Y(s)$ as

$$Y(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0}{a^n s^n + a_{n-1}s^{n-1} + \cdots + a_0}.$$

The polynomial in the denominator can be factored out as

$$Y s) = \frac{1}{a_n} \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0}{\prod_{i=1}^{n}(s - p_i)}$$

where p_i are the roots of the denominator polynomial.

Depending on p_is, we have different cases to discuss:

p_i real and distinct:

In this case, $Y(s)$ can be written as

$$Y(s) = \frac{C_1}{s - p_1} + \frac{C_2}{s - p_2} + \cdots + \frac{C_n}{s - p_n}.$$

To determine the coefficients of C_i, we can use the cover-up method. Let us multiply $s - p_i$ at both sides of the above equation. Then, we have

$$(s - p_i)Y(s) = \frac{C_1}{s - p_1}(s - p_i) + \frac{C_2}{s - p_2}(s - p_i) + \cdots + \frac{C_n}{s - p_n}(s - p_i).$$

Then we let $s = p_i$ which leads to the following formula

$$C_i = (s - p_i)Y(s)|_{s=p_i}.$$

p_i **complex:**

When the denominator polynomial contains complex roots, the complex roots appear in a pair. The partial fraction expansion includes such terms

$$\frac{C_1 s + C_2}{s^2 + as + b}.$$

To find the coefficients of C_1 and C_2, the approach of balancing the coefficients of the equation can be used. For the case of purely imaginary roots when $a = 0$, the same approach can be applied.

5.2.4 Zero and Poles

The transfer function $T(s)$ of an LTI system can be written as the ratio of two polynomials in s, that is,

$$T(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{a^n s^n + a_{n-1} s^{n-1} + \cdots + a_0} = \frac{N(s)}{D(s)}.$$

where $N(s) = b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0$ and $D(s) = a^n s^n + a_{n-1} s^{n-1} + \cdots + a_0$ are simple polynomials with real coefficients in the complex variable $s = \sigma + j\omega$. The degree n of the polynomial $D(s)$ is no less than the degree m of the polynomial $N(s)$, that is, $n \geq m$.

The roots of the numerator polynomials $N(s)$ in the complex domain are called **zeros** of the transfer function. The degree of $N(s)$ is m. Therefore, it has m roots being denoted $z_1, \ldots, z_m$ such that $N(z_i) = 0$ for $i = 1, \ldots, m$. The polynomial $N(s)$ can also be written as $N(s) = b_m \prod_{i=1}^{m}(s - z_i)$.

The roots of the denominator polynomials $D(s)$ in the complex domain are called **poles** of the transfer function. The poles of a transfer function determine the stability property, and the modes of the system response. The degree of $D(s)$ is n. Therefore, it has n roots being denoted $p_1, \ldots, p_n$ such that $D(p_i) = 0$ for $i = 1, \ldots, n$. The polynomial $D(s)$ can also be written as $D(s) = a_n \prod_{i=1}^{n}(s - p_i)$.

Therefore, the transfer function $T(s)$ can be written in the zero-pole form as

$$T(s) = K \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}$$

where $K = \frac{b_m}{a_n}$ is called the gain of the transfer function.

5.3 Stability

The concept of system stability can be characterized from two perspectives: impulse response or input-output relationship.

5.3.1 Stability from Impulse Response

A system is stable if the impulse response approaches zero as time approaches infinity. A system is unstable if the impulse response approaches infinity as time approaches infinity. A system is marginally stable if the impulse response neither decays nor grows but remains constant or oscillates. The poles of a transfer function determine the stability property of a system. Let us take a transfer function in the zero pole form for example

$$T(s) = K\frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}.$$

Using the partial fraction expansion, $T(s)$ can be written as

$$T(s) = \frac{C_1}{s - p_1} + \frac{C_2}{s - p_2} + \cdots + \frac{C_n}{s - p_n}.$$

Let us consider the case that all poles $\{p_i\}_{i=1}^{n}$ are real or complex but distinct. The inverse Laplace transform of the term corresponding to

$$\frac{C_i}{s - p_i}$$

is

$$C_i \exp(p_i t).$$

If the pole p_i is real or complex but repeated, the inverse Laplace transform is

$$tC_i \exp(p_i t).$$

Then we know, this team decays to zero when p_i is strictly in the LHP, and diverge when it is strictly in the RHP. A system is stable if and only $\exp(p_i t) \to 0$ for all p_i as $t \to \infty$, that is, the real part of p_i is negative.

Therefore, a system is stable if all poles are strictly in the LHP, and is unstable if any poles in the RHP. The system is called neutrally stable if there are non-repeated $j\omega$ poles and all other poles are in the LHP. Note that the double integrator model $1/s^2$ is unstable, and a system with repeated $j\omega$ poles is unstable. This is because the inverse Laplace transform for the term of repeated $j\omega$ poles is $t\exp(\pm j\omega t)$, which is unbounded with time.

A LTI system is stable if all the poles of the transfer function are strictly in the LHP.

5.3.2 Stability from Input-Output Relationship

A system is stable if every bounded input yields a bounded output. A system is unstable if any bounded input yields an unbounded output. The stability in this sense is called BIBO stable. The output of system can be written as the convolution of the impulse response and input signal, that is,

$$y(t) = \int_0^t h(\tau)u(t-\tau)d\tau.$$

To derive a condition for BIBO stable, let us assume that the input $|u(t)| \leq M$ is bounded, where M is a finite constant. Then, we can find a bound for the output

$$\begin{aligned} |y(t)| &= \left| \int_0^t h(\tau)u(t-\tau)d\tau \right| \\ &\leq M \int_0^t |h(\tau)|\, d\tau. \end{aligned}$$

Therefore, a system with the impulse response $h(t)$ is BIBO-stable if and only if the integral

$$\int_0^\infty |h(\tau)|\, d\tau < \infty.$$

A real-life example about the stability is the Tacoma Narrows Bridge. The original Tacoma Narrows Bridge opened on July 1st, 1940, and collapsed four months later.

5.3.3 Routh Array

Using MATLAB can easily determine the locations of all poles of a transfer function, and thus the stability of an LTI system. There are alternative ways to determine the locations of the poles without actually solving the roots of the denominator polynomial of a transfer function. Here we introduce one approach, called "Routh Array", which can determine the stability of an LTI system based on the coefficients of the denominator polynomial. Routh Array is also useful in the control design, where the controller is parameterized by several parameters. It is not easy to find the roots when there are symbolic coefficients in the polynomial. However, Routh Array is able to tell us the range of the parameters so that the closed-loop system is stable.

Before introducing the Routh stability criterion, let us discuss the necessary conditions on the coefficients of the denominator polynomial for stability of an LTI system. A necessary condition for stability is that all coefficients of the denominator polynomial are positive. Then based on the law of contrapositive, it is equivalent to say that a system is unstable if any of the coefficients of the denominator are zero or negative. Once the denominator polynomial passes the necessary condition test, then you can further inspect the coefficients to determine if the closed-loop system is stable or not.

The first step is to construct the Routh Array. The Routh Array will tell us how many poles in the LHP, in the RHP, and on the imaginary axis, but not the locations. But it is enough to determine the stability of the system. For a transfer function of with nth-degree polynomial in the denominator,

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0,$$

a table with $n+1$ arrows needs to be calculated. The first two rows are the

Row 1	a_n	a_{n-2}	a_{n-4}	$\cdots$
Row 2	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
Row 3	b_1	b_2	b_3	$\cdots$
Row 4	c_1	c_2	c_3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Table 5.1: Routh Array

coefficients of the polynomial $D(s)$. We start the calculation from the third row. For each element in the table, it needs the elements in the first column and the next column from two previous rows. Then, the four elements form a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

The element can be calculated as

$$\frac{bc - ad}{c}.$$

For example, for $i = 1, 2, 3, \ldots,$

$$b_i = \frac{a_{n-1}a_{n-2i} - a_n a_{n-(2i+1)}}{a_{n-1}}$$

and

$$c_i = \frac{b_1 a_{n-(2i+1)} - b_{i+1} a_{n-1}}{b_1}.$$

The calculations will continue until Row $n+1$. When completed, the system is stable if and only if the elements of the first column of the Routh array are all positive. The number of sign change in the first column is the number of poles in the RHP. For example, (+ - +) is counted as two sign changes. Therefore, there are two positive poles in the RHP.

5.4 Dynamic Response of an LTI System

The dynamic response of an LTI system is the output signal of the system when an input signal is applied to the system. Commonly used control input signals include impulse signal, sinusoid signal, and polynomial inputs (step signal, ramp signal, parabola, etc.). When the inputs are impulse and step signals, the outputs are called *impulse response* and *step response*, respectively. When the input is a sinusoid signal, the output in the steady state is called *frequency response.*

5.4.1 Test Signals

Step signal is a very common test input signal to control systems. Whenever a system is activated, a step signal is applied. For example, a constant speed is commanded in vehicle cruise control. It is frequently done by adding a step function to the input of the system. In addition, step response displays important characteristics of the system. Sinusoidal signal is another common input signal to control systems.

A system dynamic response is a composite response of transient response and steady-state response, that is,

$$y(t) = y_{tr}(t) + y_{ss}(t),$$

where the transient response $y_{tr}(t)$ approaches to zero when t is very large, and the steady-state value $y_{ss}(t)$ when t is very large.

5.5 Steady State Performance

5.5.1 Frequency Response

The steady state response of a system for an input sinusoidal signal is known as the frequency response.

The input signal is a sinusoidal function $u(t) = A\cos(\omega t)$. The output is the convolution integral of the input and the impulse response of the system

$$y(t) = A\int_0^t h(\tau)\cos(\omega(t-\tau))d\tau$$

Then, the integral can be written as

$$y(t) = A\int_0^\infty h(\tau)\cos(\omega(t-\tau))d\tau - A\int_t^\infty h(\tau)\cos(\omega(t-\tau))d\tau$$

The first term is called the steady-state response; and the second term will decay to zero as t grows when the system is stable. Based on the principle of superposition, we can study the steady-state response when $A = 1$. When $A \neq 1$, the steady-state response is the steady-state response when $A = 1$ scaled by A.

Using the Euler's relation:

$$u(t) = \cos(\omega t) = \frac{1}{2}\left(\exp(j\omega t) + \exp(-j\omega t)\right)$$

the steady-state response is

$$y_{ss}(t) = \frac{1}{2}\int_0^\infty h(\tau)\left(\exp(j\omega(t-\tau))\right)d\tau + \frac{1}{2}\int_0^\infty h(\tau)\left(\exp(-j\omega(t-\tau))\right)d\tau$$

Based on the definition of Fourier transform, the steady-state response can be written as

$$\begin{aligned}
y_{ss}(t) &= \frac{1}{2}\exp(j\omega t)H(j\omega) + \frac{1}{2}\exp(-j\omega t)H(-j\omega) \\
&= \mathrm{Re}(H(j\omega))\cos(\omega t) - \mathrm{Im}(H(j\omega))\sin(\omega t) \\
&= |H(j\omega)|\cos\left(\omega t + \angle H(j\omega)\right).
\end{aligned}$$

The steady-state response of a stable LTI system to a sinusoidal input is a sinusoidal with the same frequency as the input and with magnitude amplified by $|H(j\omega)|$ and the phase shifted by the angle $\angle H(j\omega)$.

Frequency Response

$$y_{ss}(t) = A|H(j\omega)|\cos\left(\omega t + \angle H(j\omega)\right)$$

We can plot the magnitude ratio and phase difference as a function of the input frequency. This is called "Bode plot", which is a frequency response of a stable LTI system. It includes a Bode magnitude plot and a Bode phase plot. Bode plots are found useful in understanding the stability of negative feedback. With bode plots, it is easy to understand the principle of signal processing. Let us consider the example of low pass filters.

Example 4 ***Passive Low Pass Filter*** *A low pass filter is a circuit that can be designed to filter out unwanted high frequencies of an electrical signal and pass the signals in the low frequency.*

5.5.2 Step Response

Step signal is a special case of the sinusoidal input $u(t) = A\cos(\omega t)$ when $\omega = 0$. Therefore, the steady-state response of a step signal can be found by setting $\omega = 0$ in the frequency response, that is,

$$y_{ss}(t) = A|H(0)|\cos\left(0t + \angle H(0)\right).$$

The steady-state response $y_s s(t)$ of a stable LTI system to a step input $u(t) = A$ is $A|H(0)|\cos\left(\angle H(0)\right)$. It is not difficult to figure out that

$$|H(0)|\cos\left(\angle H(0)\right) = H(0).$$

The value of $H(0)$ is known as "DC gain". DC gain is defined as the ratio of the steady-state output of a stable LTI system to its step inputs, that is,

$$\text{DC gain} = \lim_{s\to 0} H(s).$$

In a tracking problem, the DC gain must be one so that the tracking error in the steady-state is zero. DC gain is useful in quickly determining if the steady-state tracking error to a step signal is zero or not without using the partial fraction expansion.

Example 5 *Consider an LTI system with the transfer function*

$$H(s) = \frac{5}{s+10}.$$

Find the DC gain and determine if the steady-state tracking error is zero.

We can also obtain the steady-state step response of a stable LTI system by using the partial fraction expansion. The Laplace transform of the output is $Y(s) = H(s)/s$. The output $Y(s)$ can be written as

$$Y(s) = \frac{C_0}{s} + \frac{C_1}{s-p_1} + \cdots + \frac{C_n}{s-p_n}.$$

Since the system is table, all p_is are in the left half of the s-plane. If the inverse Laplace transform is applied, all time functions corresponding to the terms $C_i/(s-p_i)$ for $i = 1, \ldots, n$ will decay to zero, and time function of the term C_0/s is C_0. Therefore, the steady-state response of $y(t)$ is $y_{ss}(t) = C_0$. We can use the cover-up method to calculate C_0, that is,

$$C_0 = \lim_{s \to 0} sY(s)$$

Therefore, we have the final value theorem of a step response of a stable LTI system.

If an LTI system is stable, then the final value of a step response A is

$$y_{ss}(t) = \lim_{s \to 0} sH(s)\frac{A}{s} = AH(0).$$

The above result is called the “final value theorem”.

5.6 Transient Performance

5.6.1 First Order Systems

The first order system without zeros can be written as

$$H(s) = \frac{b}{s+a}.$$

In order to show the physical meaning of the parameters, it is common to write the first order system without zeros as

$$H(s) = \frac{K}{\tau s + 1},$$

where K is the DC gain, and $\tau > 0$. Therefore, the pole $-1/\tau$ is in the LHP, which means the stability of the system. Here we will discuss the transient performance of first order systems in terms of impulse response and step response.

Impulse Response

The impulse response is also called the "natural response". Rewriting the transfer function as

$$H(s) = \frac{\frac{K}{\tau}}{s + \frac{1}{\tau}},$$

the inverse Laplace transform indicates the impulse response is

$$h(t) = \frac{K}{\tau} \exp\left(-\frac{t}{\tau}\right).$$

The impulse response will decay to zero for stabile systems. To measure the rate of decay, the concept "time constant" is usually used for first order systems, which is defined as the time when the impulse response decays to $1/e$ of the initial value, and e is known as Euler's number approximately equal to 2.71828. If we draw a tangent line at the initial time, and the slope of the line is $-k/\tau^2$ since

$$\frac{d}{dt}\frac{K}{\tau} \exp\left(-\frac{t}{\tau}\right)\bigg|_{t=0} = -\frac{K}{\tau^2}.$$

This straight line intersects with the $y = 0$ line at $t = \tau$. Therefore, the time constant measures the rate of decay.

Step Response

Since the Laplace transform of the unit step signal is $1/s$, the Laplace transform of the step response is

$$Y(s) = \frac{K}{s(\tau s + 1)},$$

Using the partial fraction expansion

$$Y(s) = \frac{K}{s} - \frac{K}{s + \frac{1}{\tau}},$$

and the inverse Laplace transform, the step response is

$$y(t) = K - K \exp\left(-\frac{t}{\tau}\right)$$

The first term originates in the pole of the input and is called the forced response; since this term does not decay to zero with increasing time, it is also called the steady-state response. The second term originates in the pole of the transfer function and is called the natural response; since this terms decay to zero with increasing time, it is also called the transient response. The time constant in the step response can be described as the time for the response to rise to 63% of its final value. At two time constants, the response reaches 86.5% of its final value. At $t = 3\tau$, 4τ, and 5τ, the response reaches 95%, 98.2%, and 99.3%, respectively, of its final value. As we see here, the time constant is a metric of the speed of the system response. After about 5 time constants 5τ, the response remains within 1% of its final value, and is considered that the response is at steady-state.

Example 6 *Consider the transfer function*

$$H(s) = \frac{5}{s+2}$$

What is the steady-state value, and when the unit step response reaches steady state?

What is the use of all the observations above? It is to identify the parameters K and τ in a transfer function when it is not available. To do so, we can use a unit step signal to test the system, and plot the step response. Let us see this example blew.

Example 7 *The unit step response of a first order system is shown in Fig. 5.7. What is the DC gain K and time constant τ in the standard first order transfer function?*

For step responses, it is also common to define the transient performance using settling time, overshoot, and rise time. In the following, we will give the definitions and calculate their values for first order systems.

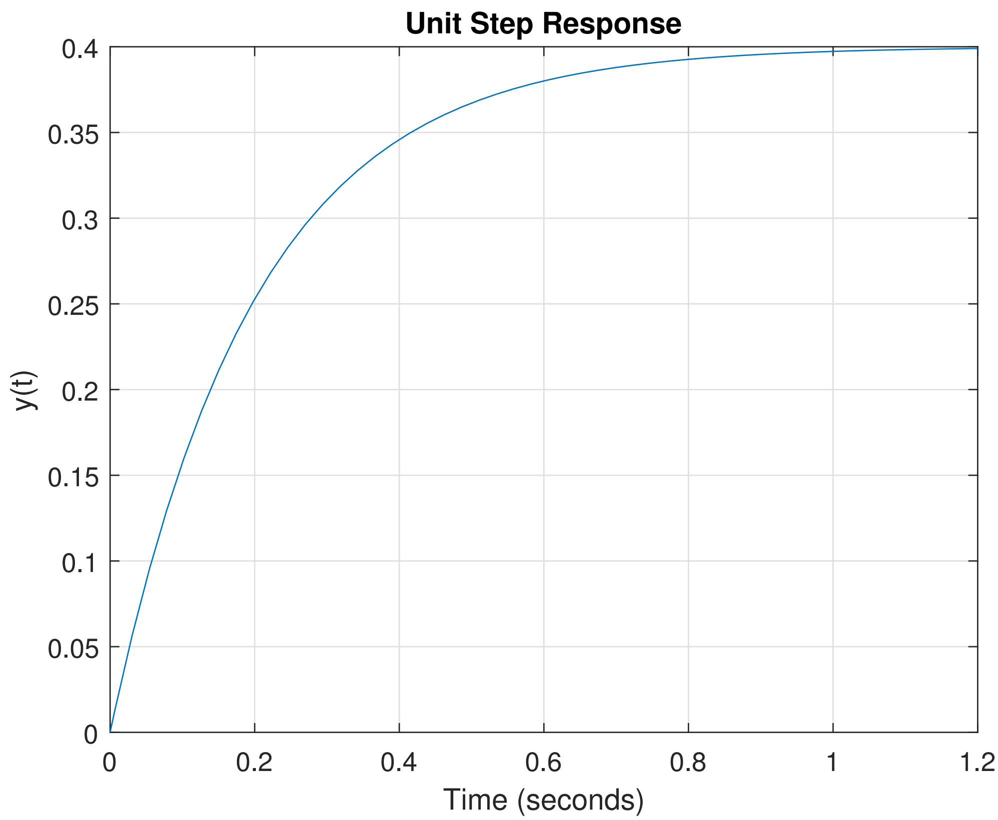


Figure 5.7: Unit Step Response Test

- **Settling Time**: Time for the response to reach and stay within 1% of its final value, denoted by t_s. According to the definition, we have

$$K - K\exp\left(-\frac{t_s}{\tau}\right) = 0.99K.$$

By solving the above equation for t_s, we find the settling time

$$t_s = 2\ln(10)\tau \approx 4.6\tau.$$

- **Rise Time**: Time for the response to go from 10% to 90% of its final value, denoted by t_r. According to the definition, we have $K - K\exp\left(-\frac{t_{0.9}}{\tau}\right) = 0.9K$ and $K - K\exp\left(-\frac{t_{0.1}}{\tau}\right) = 0.1K$. The rise can be found as

$$t_r = [\ln(0.9) - \ln(0.1)]\tau \approx 2.2\tau.$$

Damping ratio	System response
$\zeta = 0$	undamped
$0 < \zeta < 1$	underdamped
$\zeta = 1$	critically damped
$\zeta > 1$	over-damped

Table 5.2: Second-order system response

- **Overshoot**:the maximum amount the system overshoots its final value divided by its final value, denoted by PO. For first order systems, the percentage overshoot is $PO = 0\%$.

5.6.2 Second Order Systems

A standard second-order transfer function with unity DC gain can be written as

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

Depending the value of ζ, the system response shows different behaviors as shown in Table 5.2.

Real Roots

When $\zeta > 1$, the transfer function $H(s)$ has two distinct real roots. Therefore, it can be written in the following form

$$H(s) = \frac{\alpha_1\alpha_2}{(s+\alpha_1)(s+\alpha_2)}.$$

Let us assume $\alpha_2 > \alpha_1 > 0$. The transfer function has two poles at $-\alpha_1$ and $-\alpha_2$. A partial fraction expansion of $H(s)$ indicates that the impulse response contains two components

$$H(s) = \frac{C_1}{s+\alpha_1} + \frac{C_2}{s+\alpha_2},$$

where C_1 and C_2 are determined by the β in the numerator, and they determine the sizes of each components. The inverse Laplace transform results in the impulse response

$$h(t) = C_1 \exp(-\alpha_1 t) + C_2 \exp(-\alpha_2 t).$$

The impulse response of a second order system can be regarded as a weighted sum of the impulse responses of two first order systems. The locations of the poles determine the components and the numerator determines the size of each components. This claim also applies to higher order complex systems. The component $\exp(-\alpha_2 t)$ decays faster than the component $\exp(-\alpha_1 t)$ since $\alpha_2 > \alpha_1$. Then, we can say $-\alpha_2$ is a faster pole than $-\alpha_1$. In general, a pole which is further to the left in the s-plane are associated with faster components in the impulse response than those associated with poles closer to the imaginary axis.

Example 8 *Find the impulse response of the transfer function*

$$H(s) = \frac{2}{s^2 + 3s + 2}.$$

The denominator is $D(s) = s^2 + 3s + 2 = (s+1)(s+2)$*. The poles of* $H(s)$ *are at* $s = -1$ *and* $s = -2$*. A partial fraction expansion of* $H(s)$ *results in*

$$H(s) = \frac{2}{s+1} - \frac{2}{s+2}.$$

The inverse Laplace transform gives the time function

$$h(t) = 2\exp(-t) - 2\exp(-2t).$$

The impulse response is shown in Fig. 5.8. At the beginning, the component $\exp(-2t)$ *decays fasters, and therefore the difference between the two component becomes large and large. After some point, the two components decays to zero and the difference between them becomes smaller and smaller. Eventually, the overall response decays to zero as the system is stable.*

Complex Roots

As in the first order system case, let us write the second order system with a pair of complex poles in the form

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

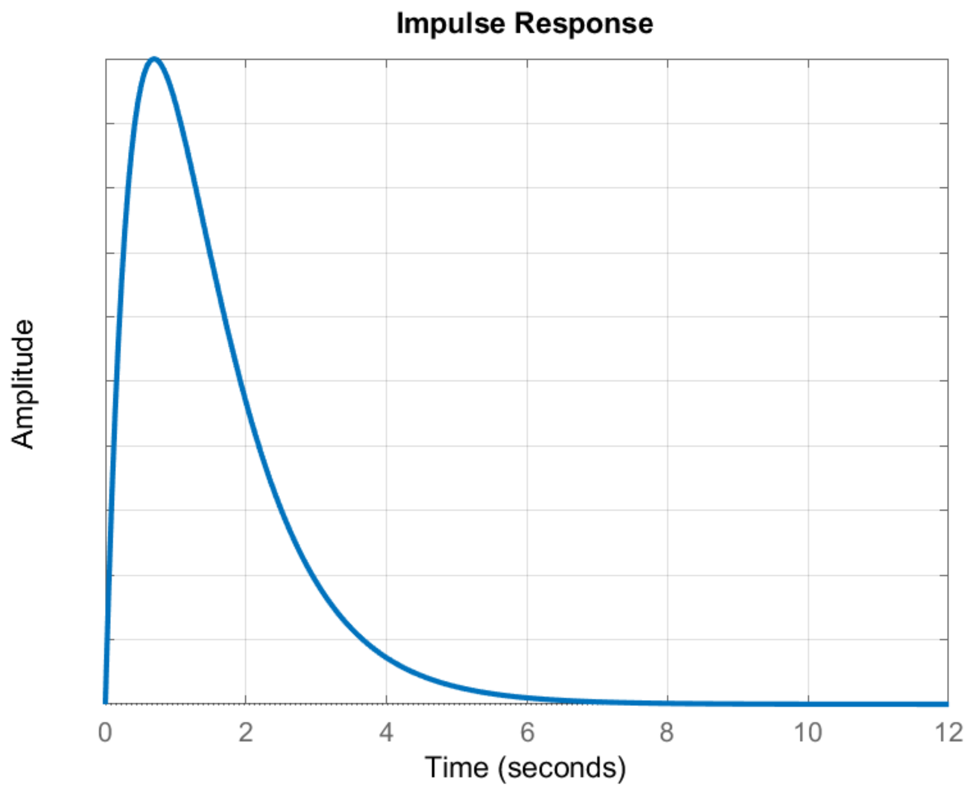


Figure 5.8: Impulse Response

so that it is easy to identify the physical meaning of the coefficients $0 < \zeta < 1$ and $\omega_n > 0$, where ζ and ω_n are defined to be the "damping ratio" and "undamped natural frequency", respectively. First, let us determine the location of the poles in the s-plane. Let us write the denominator of the polynomial as $s^2+2\zeta\omega_n s+\zeta^2\omega_n^2+(1-\zeta^2)\omega_n^2 = (s+\sigma)^2+w_d^2$, where $\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Here ω_d is called "damped natural frequency". Let $(s+\sigma)^2+w_d^2 = 0$. We obtain a pair of complex poles at $s = -\sigma \pm j\omega_d$, where $-\sigma$ is the real part of the complex poles, and $\pm\omega_d$ are the imaginary parts of the complex poles. Therefore, ω_n is the maganitude of the complex poles, and the phase is $\arcsin(\zeta)$ for one of the poles.

When $\zeta = 0$, the transfer function becomes

$$H(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

The inverse Laplace transform shows that

$$h(t) = \frac{1}{\omega_n} \sin(\omega_n t).$$

In this case, we have a pair of imaginary poles at $\pm j w_n$, and the impulse response is a sinusoid with frequency ω_n. Therefore, there are no damping. That is the reason why ω_n is called "undamped natural frequency". With the increase of ζ, the poles move away from the imaginary axis to the left. The impulse response is

$$h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} \exp(-\sigma) \sin(\omega_d t).$$

Even though the response will oscillate, it will decay to zero eventually. Therefore, ω_d is called "damped natural frequency". The variable $1/\sigma$ is also defined as the time constant. After four time constants, it is general considered that the system is at the steady-state.

The unit step response can be found by the inverse Laplace transform of $Y(s)$, that is,

$$Y(s) = H(s)\frac{1}{s}.$$

The time function of the step response is thus

$$y(t) = 1 - \exp(-\sigma t) \left(\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d) \right)$$

Use the trigonometric function (2.1),

$$\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d t) = \sqrt{1 + \frac{\sigma^2}{\omega_d^2}} \cos \left(\omega_d t - \arctan \left(\frac{\sigma}{\omega_d} \right) \right).$$

Using the definition $\sigma = \zeta \omega_n$ and $\omega_d = \omega_n \sqrt{1-\zeta^2}$, we have

$$\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d) = \frac{1}{\sqrt{1-\zeta^2}} \cos\left(\omega_d t - \arcsin(\zeta)\right)$$

since

$$\arctan\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = \arcsin(\zeta).$$

Finally, we can obtain the step response in a different form

$$y(t) = 1 - \frac{\exp(-\sigma t)}{\sqrt{1-\zeta^2}} \cos\left(\omega_d t - \arcsin\left(\zeta\right)\right).$$

Performance specifications for a control system design often involve certain requirements associated with the time response of the system. We can characterize some key features of the step response of second order systems used for control design. The requirements for a step response are expressed in terms of the standard quantities: rise time, overshoot, and settling time. These performance specifications have the same definitions as those of the first order system case. Because of their importance, we would like to define them again:

- Rise time (t_r): The time it takes for the response to rise from 10% to 90% of its steady-state value.

  Unlike the first order system case, it is difficult to give an analytical solution of the rise time t_r in terms of the parameters ω_n and ζ. Since ζ is a value between 0 and 1, we can use the average value of $\zeta = 0.5$ and the rise time is roughly

$$t_r \approx \frac{1.8}{\omega_n}.$$

- Peak time (t_p): The time it takes for the response to reach the maximum overshoot point.

  We can find the peak time by taking the time derivative of $y(t)$, and let the derivative to be zero. The time derivative of $y(t)$ is

$$\begin{aligned}
y^{(1)}(t) =& \sigma \exp(-\sigma t) \left(\cos(\omega_d t) + \frac{\sigma}{\omega_d} \sin(\omega_d t)\right) \\
&+ \exp(-\sigma t) \left(\omega_d \sin(\omega_d t) - \sigma \cos(\omega_d t)\right) \\
=& \exp(-\sigma t) \left(\frac{\sigma}{\omega_d} + \omega_d\right) \sin(\omega_d t)
\end{aligned}$$

All the peaks points correspond to $y^{(1)}(t) = 0$, $y^{(1)}(t-) > 0$, and $y^{(1)}(t+) < 0$. Therefore, we know that $w_d t = k\pi$ for $k = 1, 2, \dots$ and the $\omega_d t_p = \pi$. Therefore, the peak time

$$t_p = \frac{\pi}{\omega_d}.$$

- Overshoot (OP): the maximum amount the system overshoots its final value divided by its final value.

  The overshoot can be calculated using the peaking value $y(t_p)$ at the peak time t_p, where

  $$y(t_p) = 1 - \frac{\exp(-\sigma t_p)}{\sqrt{1-\zeta^2}} \cos\left(\omega_d t_p - \arcsin\left(\zeta\right)\right).$$

  It can be calculated that

  $$\exp(-\sigma t_p) = \exp\left(-\zeta\omega_n \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}\right) = \exp(-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}),$$

  and

  $$\cos\left(\omega_d t_p - \arcsin\left(\zeta\right)\right) = \cos\left(\omega_d \frac{\pi}{\omega_d} - \arcsin\left(\zeta\right)\right) = -\sqrt{1-\zeta^2}.$$

  Then, we have

  $$y(t_p) = 1 + \exp(-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}).$$

  Since the DC gain $H(0)$ is 1. The final value of $y(t)$ is 1. Therefore, we have

  $$OP = 100\exp(-\pi\frac{\zeta}{\sqrt{1-\zeta^2}})\%.$$

- Settling time (t_s): The time it takes for the response to reach 1% of the steady-state value and stay within the 1% range.

  This means that $0.99 < y(t) < 1.01$ for $t \geq t_s$, that is,

  $$0.99 < 1 - \frac{\exp(-\sigma t)}{\sqrt{1-\zeta^2}} \cos\left(\omega_d t - \arcsin\left(\zeta\right)\right) < 1.01.$$

This is equivalent to say that

$$\left|\frac{\exp(-\sigma t)}{\sqrt{1-\zeta^2}}\cos\left(\omega_d t - \arcsin\left(\zeta\right)\right)\right| < 0.01$$

To obtain an analytical expression for the setting time in which

$$\left|\frac{\exp(-\sigma t_s)}{\sqrt{1-\zeta^2}}\cos\left(\omega_d t - \arcsin\left(\zeta\right)\right)\right| \leq 0.01$$

is really hard. Since the response is bounded by the envelop determined by the exponential term $\exp(-\sigma t)$, we then borrow the concept of time constant. According to the definition of time-constant,

$$\tau = \frac{1}{\sigma}.$$

Similar to the first order case, to decay within the 1% range of steady-state value, the settling time is

$$t_s \approx \frac{4.6}{\sigma}.$$

5.6.3 Higher Order Systems

Effects of Zeros

Now let us assume we have a transfer function $H_1(s)$ and the corresponding response $Y_1(s) = H_1(s)U(s)$ is $y(t)$ for an input $U(s)$. Now let us consider another transfer function

$$H_2(s) = \frac{s+z}{z}H_1(s).$$

The transfer function has an addition zero compared with the transfer function $H_1(s)$. The output of the second system with the same input is $Y_2(s) = H_2(s)U(s)$, where $Y_2(s)$ can be written as

$$Y_2(s) = H_1(s)U(s) + \frac{s}{z}Y_1(s).$$

By taking the inverse Laplace transform and using the property (2.2), we have

$$y_2(t) = y_1(t) + \frac{1}{z}y_1^{(1)}(t)$$

by assuming that $y_1(0) = 0$.

When there is an extra zero, the response consists of two parts: the original response (the response without an extra zero) and the weighted derivative of the original response. When z is very large, the zero has little effect to the original response. When z is small, the zero has substantial influence to the original response.

For step responses, the derivative is typically positive at the start of a step response. The positive part will be added to the original step response. Therefore, the major effect of zeros to step response is to increase overshoot and reduce rise time.

It is interesting when the zeros are at the RHP. Zeros at RHP are also called "nonminimum-phase zeros". The major effect of nonminimum-phase zeros is the undershoot. The step response will start towards an opposite direction from the reference input. This may make the response slow.

Effects of Poles

Additional poles may increase the rise time.

Chapter 6

Design of Control Systems

The learning outcomes of this chapter are to

- Find steady-state error for a unity-feedback system;
- Be aware of the central role of error signals in design of control systems;
- Recognize the improvements afforded by feedback control in reducing system sensitivity to parameter changes, disturbance rejection, and measurement noise attenuation;
- Be familiar with the PID controller as a key element of many feedback systems.

This chapter focuses on the controller design in the closed-loop system. A good control design should include but not limited to the following desired characteristics:

- **Stability:** The control system must be stable all the time. It is a basic and absolute requirement.
- **Tracking:** The system output should be as close as possible to the desired input signals, including both the desired transient performance and a small steady-state tracking error.
- **Disturbance rejection and noise attenuation:** All systems have unwanted disturbance inputs, which are usually constant signals, such as bias. The system should not response significantly to the disturbance input. The control design should have the ability to reject the effects

of a constant disturbance. Also the sensing noises are inevitable. A good control design should attenuate the effect of sensing noises.

- **Robustness:** It is almost impossible to obtain an accurate model of a plant. The characteristics of the control system should be insensitive to the parameter changes in the plant due to environmental changes, such as temperature, humidity, altitude. The control performance should not be comprised even the design is based on inaccurate parameters in the plant.

Most developments in the following are based on the unity-feedback structure. Most simple feedback system can be transformed in to the unity feedback case as shown in Fig. 6.1. The plant $G(s)$ includes the physical process, power amplifiers, actuators, gears and so on. The variable $Y(s)$ is the known as the controlled variable or the system output. It is the one we want to control. The variable $R(s)$ in the unity feedback is the desired value of the system output. The variable $U(s)$ is the control signal. The difference between them is the system error $E(s) = R(s) - Y(s)$. The unit feedback can be used when the sensor can be modeled as a pure gain.

6.1 What is a good control design?

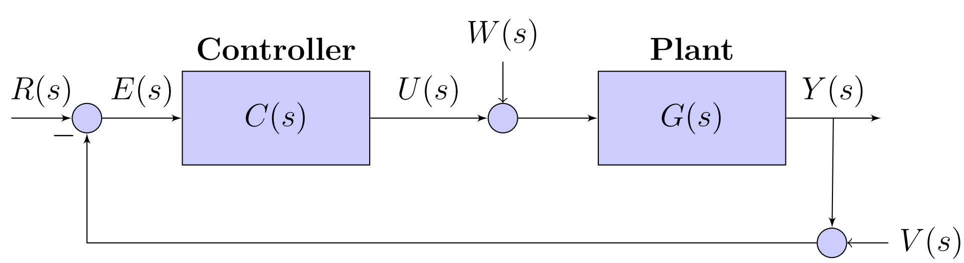


Figure 6.1: Unity Feedback

Let us first derive the basic transfer functions from various inputs to the output. Define the *sensitivity function* and *complementary sensitivity function* as

$$\mathcal{S} = \frac{1}{1 + C(s)G(s)}$$

and

$$\mathcal{T} = \frac{C(s)G(s)}{1 + C(s)G(s)},$$

respectively. It is easy to see that $\mathcal{S} + \mathcal{T} = 1$.

The transfer function from the reference input $R(s)$ to the output $Y(s)$ is

$$H_r(s) = \mathcal{T},$$

the transfer function from the disturbance input $W(s)$ to the output $Y(s)$ is

$$H_w(s) = G(s)\mathcal{S},$$

and the transfer function from the sensor noise $V(s)$ to the output $Y(s)$ is

$$H_v(s) = -\mathcal{T}.$$

Based on the principle of superposition,

$$Y(s) = \mathcal{T}R(s) + \mathcal{S}G(s)W(s) - \mathcal{T}V(s).$$

The error signal is

$$E(s) = R(s) - Y(s) = \mathcal{S}R(s) - \mathcal{S}G(s)W(s) + \mathcal{T}V(s).$$

6.1.1 Stability

Let us assume that the controller $C(s)$ and the plant $G(s)$ can be written as

$$C(s) = \frac{c(s)}{d(s)}$$

and

$$G(s) = \frac{b(s)}{a(s)},$$

respectively. The characteristic equation is

$$a(s)d(s) + b(s)c(s) = 0.$$

Even though the plant $G(s)$ may be unstable, we can design the controller by choosing $c(s)$ and $d(s)$ to make the closed-loop system stable.

6.1.2 Tracking

The tracking problem is to design the controller $C(s)$ so that the output $Y(s)$ follows the input $R(s)$ as close as possible. To make the steady-state tracking error zero when the input is a step signal, we can verify the DC gain to see if $\mathcal{T}(0) = 1$ or $\mathcal{S}(0) = 0$. For other types of inputs, more detailed will be given later.

6.1.3 Regulation

The regulation problem is to tracking a constant reference input as close as possible regardless of the disturbance inputs and sensing noises. From the basic equation, we know

$$E(s) = \mathcal{S}R(s) - \mathcal{S}G(s)W(s) + \mathcal{T}V(s).$$

To reduce the effects of disturbances and noises, we need the $\mathcal{S}$ and $\mathcal{T}$ to be "small" in some sense. It seems to be a conflicting requirement to make both the sensitivity and the complementary sensitivity function "small" at the same time. The make the sensitivity function "small", we need to design a controller $C(s)$ so that $C(s)G(s)$ is "large". However, when $C(s)G(s)$ is "large", the complementary sensitivity function $\mathcal{T}$ is close to 1. When $C(s)G(s)$ is "small", the complementary sensitivity function $\mathcal{T}$ is "small". However, the sensitivity function $\mathcal{S}$ is closed to 1. The solution to this dilemma is to explore the frequency property of the disturbance and noises. The plant disturbances occur at very low frequency, such as a bias. But the sensing noises occur often at the high frequency. Therefore, we could design the controller $C(s)$ so that the magnitude $|C(j\omega)|$ is large at the low frequency and small at the high frequency.

6.1.4 Robustness

It is important that the controller parameters are chosen in such a way that the closed-loop system is not sensitive to variations in process dynamics. Parameter changes due to heat or other causes should not appreciably affect the performance of a system. The general topic of system characteristics changing with system parameter variations is called "sensitivity". The sensitivity of a transfer function G to one of its parameters K is defined as the ratio of percentage change in G to percentage change in K as

$$\mathcal{S}_K^G = \frac{dG/G}{dK/K} = \frac{K}{G}\frac{dG}{dK}.$$

First, let us calculate the sensitivity of the closed-loop transfer function $T(s)$ to the plant transfer function $G(s)$. The closed-loop transfer function is

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}.$$

According to the definition,

$$\mathcal{S}_T^G = \frac{G}{T}\frac{dT}{dG} = \frac{G}{\frac{CG}{1+CG}}\frac{(1+CG)C - C^2G}{(1+CG)^2} = \frac{1}{1 + G(s)C(s)}.$$

This is exactly how sensitivity function is defined, and it is the transfer function from the reference input to the system error.

6.2 Tracking Problems

6.2.1 Test Signals

The common test signals are steps signals, ramp signals, and parabolic inputs shown in Fig. 6.2. When sensors have their own dynamics, the units of the input and the output are not the same in general. The unit of the input and the sensor output is typically voltage. The output could be position, temperature, and angles. The test signal can be mathematically written as polynomial inputs as

$$r(t) = \frac{t^k}{k!},$$

where k is the order of the polynomial input, t is the time variable, and $k!$ is the factorial of k.

6.2.2 System Types for Reference Input

Control systems can be classified according to their ability to tracking polynomial inputs.

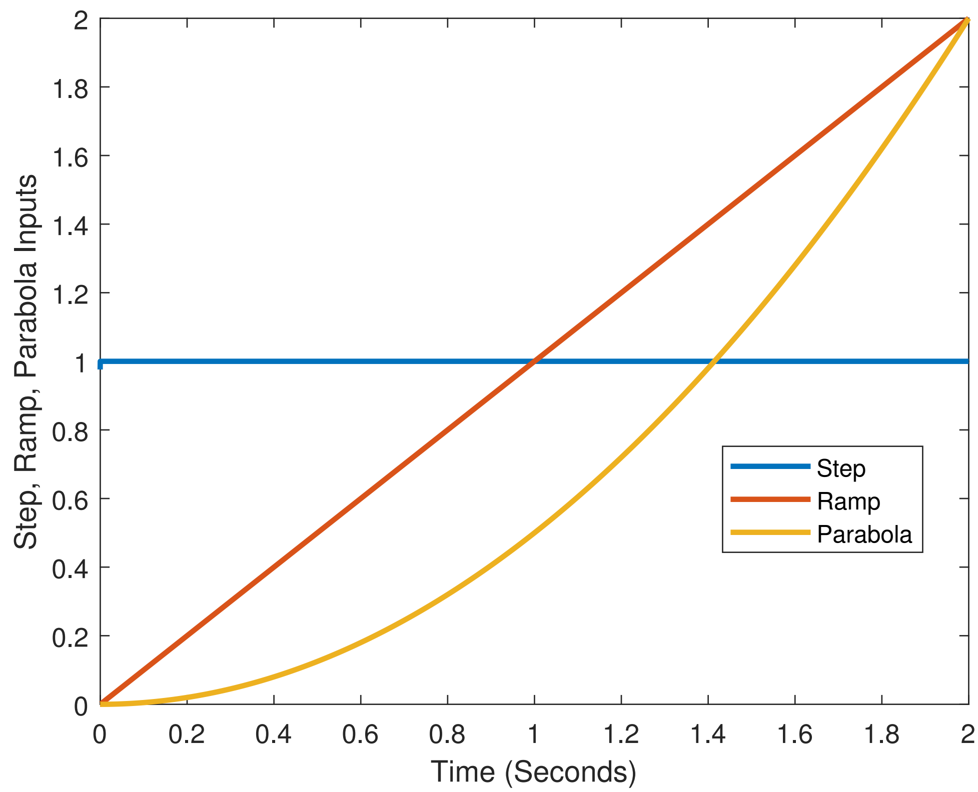


Figure 6.2: Test Signals

According to the block diagram shown in Fig. 6.3, the tracking error can be written as

$$E(s) = R(s) - Y(s).$$

Let us assume that $W(s) = 0$ and $V(s) = 0$. The transfer function from $R(s)$ to $Y(s)$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{CG}{1 + CGH}$$

implies that $E(s) = R(s) - H(s)R(s) = (1 - T(s))R(s)$. The Laplace transform of a polynomial with degree k, that is, $r(t) = t^k/k!$, is

$$R(s) = \frac{1}{s^{k+1}}.$$

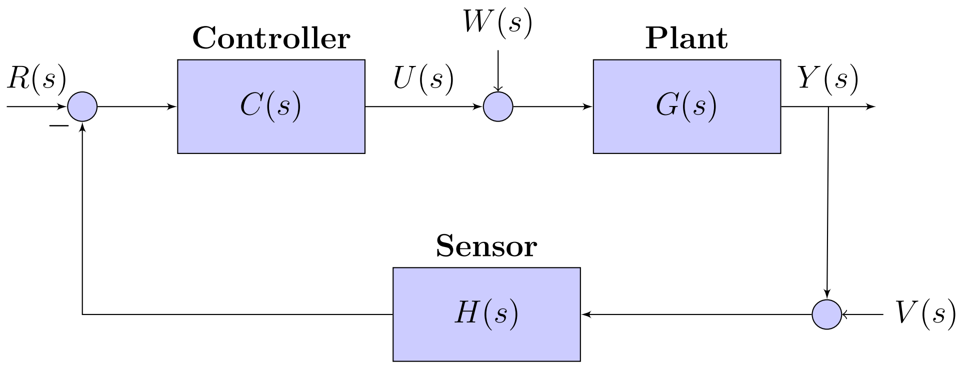


Figure 6.3: Feedback Control System with Sensor Dynamics

The system is type n if it can track a polynomial of degree n with a constant error. It turns out that a system is type n if $1 - H(s)$ can be written as $1 - T(s) = s^n P(s)$ and $P(0) \neq 0$ is a finite constant. In other words, the number of zeros of the transfer function from the reference input to the system error determines the type of the system for reference input. To find the steady-state tracking error with the assumption that the closed-loop system is stable, we can use the final value theorem. The steady-state tracking error e_{ss} is

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s(1 - H(s)) \frac{1}{s^{k+1}} = \lim_{s \to 0} \frac{1 - T(s)}{s^k} = \lim_{s \to 0} \frac{s^n}{s^k} P(s).$$

When $n > k$, $e_{ss} = 0$. When $n < k$, $e_{ss} = \infty$. When $n = k$, $e_{ss} = P(0)$.

When the closed-loop control system is a unity feedback, there is a simple way to determine the system type by check the number of poles of the forward path gain $C(s)G(s)$. Let us write $C(s)G(s)$ as the form $\frac{Q(s)}{s^n}$ and $Q(0) \neq 0$ is a finite constant. The constant $Q(0)$ is called the error constant to quantify the tracking error. It turns out the system is type n. Let us assume the reference input is polynomial of degree n. Then the Laplace transform is $R(s) = 1/s^{n+1}$. The tracking error is

$$e_{ss} = \lim_{s \to 0} s \frac{1}{s^{n+1}} \frac{1}{1 + \frac{Q(s)}{s^n}} = \lim_{s \to 0} \frac{1}{s^n} \frac{s^n}{s^n + Q(s)}.$$

For the step response, $n = 0$,

$$e_{ss} = \frac{1}{1 + Q(0)},$$

and $Q(0)$ is called the position error. For the ramp input, $n = 1$,

$$e_{ss} = \frac{1}{Q(0)},$$

and $Q(0)$ is called the velocity constant. When $n = 2$,

$$e_{ss} = \frac{1}{Q(0)},$$

and $Q(0)$ is called the acceleration constant.

6.2.3 System Types for Disturbance Input

To study the disturbance rejection, we can assume $R(s) = 0$ because of the principle of superposition of linear systems. In this case,

$$E(s) = -Y(s) = \frac{G(s)}{1 + C(s)G(s)H(s)} W(s),$$

and the transfer function from $W(s)$ to $E(s)$ is

$$H_w(s) = -\frac{G(s)}{1 + C(s)G(s)H(s)}.$$

We can also classify control systems according to their abilities to reject disturbances. Let us assume that the disturbance is the form of polynomials, that is,

$$W(s) = \frac{t^k}{k!}.$$

A control system is called type k for the disturbance input if k is the highest order of the polynomials that the control system can reject with a constant non-zero steady-state error. We can use the final value theorem to calculate the steady-state error

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sH_w(s) \frac{1}{s^{k+1}} = \lim_{s \to 0} \frac{H_w(s)}{s^k}.$$

When the disturbance is a constant, then $k = 0$. In this case,

$$e_{ss} = \lim_{s \to 0} H_w(0).$$

Typical examples include constant wind against an antenna control system, or the constant heat loss in a temperature control system. The system is type 0 if $H_w(0) \neq 0$ is a constant. When the disturbance is a ramp function, then

$$e_{ss} = \lim_{s \to 0} \frac{H_w(s)}{s}.$$

The system is called type 1 when

$$\lim_{s \to 0} \frac{H_w(s)}{s}$$

is a nonzero constant. In general, a system type to disturbance input can be defined according to the number of zeros at $s = 0$ of the transfer function $T_w(s)$. Suppose that a system is type n. If $n > k$ with k being the degree of the disturbance input, then $e_{ss} = 0$; if $n < k$, the error is unbounded; when $n = k$, the error is a nonzero constant.

6.3 PID Controller

The PID controller has three terms in the form of

$$D(s) = K_P + \frac{K_I}{s} + K_D s,$$

where K_P is the proportional gain, K_I is the integral gain, and K_D is the derivative gain.

6.3.1 P Controller

The proportional term can be used alone, and the controller is called "P-Controller". The output of the P-controller is proportional to the error signal $e(t) = r(t) - y(t)$, and the output is $u(t) = K_P e(t)$. The transfer function of the P-Controller is a gain K_P. Assume that the plant has the transfer function

$$G(s) = \frac{b(s)}{a(s)},$$

and the system is in a unity feedback structure. The characteristic equation is $a(s) + K_P b(s) = 0$. The P-Controller will not change the type of the system. Assume that the plant is type 0, then the steady-state tracking error is $e_{ss} = \frac{1}{1+K_P G(0)}$. It can be seen that the large K_P, the smaller the tracking error.

Consider the standard first order system

$$G(s) = \frac{K}{\tau s + 1}$$

and second order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

The type of the system are 0 and 1 for the first order system and the second order system, respectively. The steady-state tracking error for a unit step signal are

$$e_{ss} = \frac{1}{1 + KK_P}$$

and

$$e_{ss} = \frac{1}{1 + K_P}.$$

We can conclude that the larger K_P, the smaller the steady-state tracking error. For the transient performance, the closed-loop transfer function for first order plant is

$$H(s) = \frac{KK_P}{\tau s + 1 + KK_P} = \frac{\frac{KK_P}{1+KK_P}}{\frac{\tau}{KK_P}s + 1}.$$

We can see that the larger K_P, the smaller the time constant, and the better the transient performance. For second-order systems, the characteristic equation is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 + K_P\omega_n^2.$$

The natural frequency for the closed-loop system becomes $\sqrt{K_P(1+\omega_n^2)}$ and the damping ratio becomes

$$\zeta\frac{\omega_n}{\sqrt{K_P(1+\omega_n^2)}}.$$

Even though a large K_P will reduce the steady-state tracking error, it also reduce the dampling ratio. Therefore, the transient response will have a large overshoot for a large K_P. A large K_P can also reduce the rise time.

The analysis for the disturbance input is similar. One way to reduce the steady-state tracking error without using a very large gain is to introduce the "I-controller", which will be discussed below.

6.3.2 I Controller

The integral controller has the form

$$u(t) = K_I \int_0^t e(\tau) d\tau,$$

and the Laplace transform of the "I controller" is

$$D(s) = \frac{K_I}{s}.$$

The output of the I controller is linear proportional to the integral of system errors. The I controller has its own dynamics with one integrator. The purpose of the I controller is to reduce the steady-state errors, and improve the steady-state output response to disturbances.

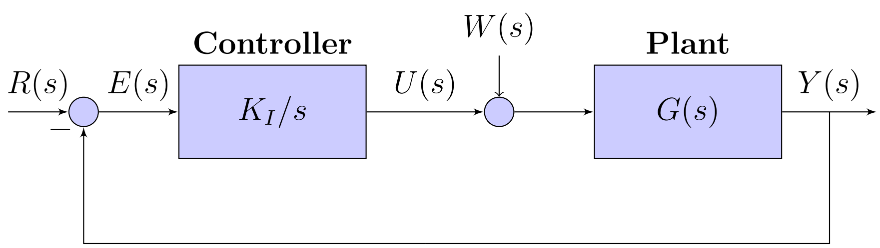


Figure 6.4: Unity Feedback

Assume that the plant has the transfer function

$$G(s) = \frac{b(s)}{a(s)},$$

and the system is in a unity feedback structure. Then, the characteristic equation of the closed-loop system is $sa(s) + K_I b(s) = 0$. For first order systems, the characteristic equation is

$$\tau s^2 + s + K_I K = 0.$$

We can adjust K_I to improve the dynamic response. For second order systems, the characteristic equation is

$$s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + K_I \omega_n = 0.$$

Row 1	1	ω_n^2
Row 2	$2\zeta\omega_n$	$\mathrm{K}_I\omega_n$
Row 3	b_1	0
Row 4	$K_I\omega_n$	

Table 6.1: Routh Array

Based on Routh array shown in Table 6.1, we have

$$b_1 = \frac{2\zeta\omega_n^2 - K_I\omega_n}{2\zeta\omega_n} = \frac{2\zeta\omega_n - K_I}{2\zeta}.$$

It is easy to see that the system may be unstable if K_I is too large.

The forward path transfer function is $G(s)K_I/s$. Therefore, the I-controller increases the system type by 1. For first order systems,

$$\frac{KK_I}{s(\tau s + 1)}.$$

The system is type 1. It can track step reference inputs with a zero steady-state error. The same claim applies to standard second-order systems. For unit step response, the transfer function from the step reference input to the control input of the standard first and second order systems is

$$\frac{U(s)}{R(s)} = \frac{K_I/s}{1 + K_I G(s)/s} = \frac{K_I}{s + K_I G(s)}.$$

In the steady state,

$$u_{ss} = \lim_{s\to 0} \frac{K_I}{s + K_I G(s)} = \frac{1}{G(0)}.$$

It makes sense that in the steady-state the control signal is $1/G(0)$ since the product of the control signal and the DC gain of the process is 1, which is the output at steady state.

The transfer function from the disturbance input to the error is

$$\frac{E(s)}{W(s)} = \frac{-G(s)}{1 + G(s)K_I/s} = -\frac{sG(s)}{s + G(s)K_I}.$$

For standard first and second order processes, the system is type 1 for the disturbance input. The system has zero steady-state output error for step disturbance inputs. The transfer function from the disturbance input to the control signal is

$$\frac{U(s)}{W(s)} = \frac{-G(s)K_I/s}{1+G(s)K_I/s} = -\frac{G(s)K_I}{s+G(s)K_I}.$$

When the disturbance is a unit step signal, the steady-state control signal is

$$u_{ss} = \lim_{s\to 0} -\frac{G(s)K_I}{s+G(s)K_I} = -1.$$

It makes sense that the sum of the control signal and the disturbance is zero so that the disturbance has no influence on the output.

The conclusion is that in this case, integral feedback results in zero steady-state error for both reference input and disturbance input. Furthermore, plant parameter changes can be tolerated; that is, the results above are independent of the plant parameter values. Also, regardless of the integral gain K_I, the asymptotical tracking, and disturbance rejection properties are preserved, provided that the closed-loop system remains stable. These properties of integral control are referred to as robust.

6.3.3 D Controller

The derivative feedback is also called rate feedback. It has the form

$$u(t) = K_D e^{(1)}(t)$$

in the time domain. The control signal is proportional to the rate of the error system. The Laplace transform of the D controller is $U(s) = K_D sE(s)$. The D-control cannot be used alone since it does not provide the necessary control signal at the steady-state since the derivative of steady-state signal is always zero. The goal of the D controller is to improve closed-loop stability, speed up transient response, and reduce overshoot. The disadvantage of D controller is to amplify noise.

6.4 PID Controller

The PID control is the one which contains all three terms. In the time domain, the controller in the parallel form is

$$u(t) = K_P e(t) + K_I \int_{t_0}^{t} e(\tau) d\tau + K_D e^{(1)}(t).$$

By properly tuning all the three parameters, a PID controller has all advantages of each individual term.

6.5 PID Controller Tuning

The PID controller can be written in different forms: parallel form and standard form, as

$$D(s) = K_P + \frac{K_I}{s} + K_D s$$

and

$$D(s) = K_P(1 + \frac{1}{T_I s} + T_D s),$$

respectively.

6.5.1 The Step Response Method

The step response approach is to test a process using a step signal in the open loop. We assume no knowledge of the plant or process information. This method can be viewed as a traditional method based on modeling and control where a very simple process model is used. The step response is characterized by a first order plus time delay model in the form of

$$H(s) = \frac{A \exp(-\tau_d s)}{\tau s + 1},$$

as shown in Fig. 6.5, where τ_d is a time delay, A is the DC gain, and τ is the time constant. The DC gain A can be determined by the steady-state value of the unit step response. The time delay is time when the input is applied until the output starts to response to the input. The time constant can be calculated using the concept of rise time, 2.2τ, which is the time for

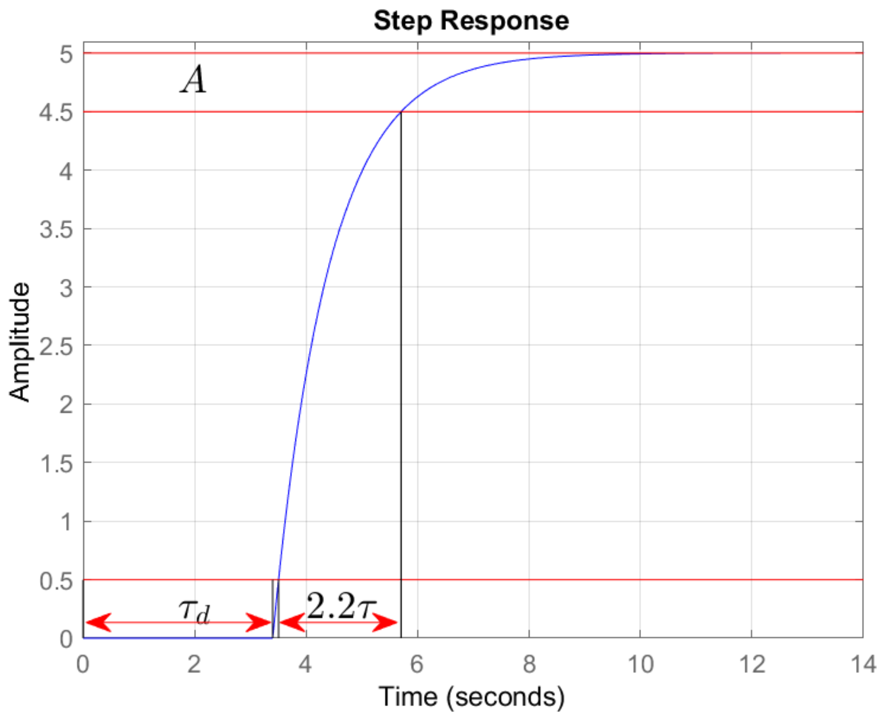


Figure 6.5: Ziegler-Nichols Tuning

the output to raise from 10% to 90% of its final value. Then, we calculate the reaction rate

$$R = \frac{A}{\tau}.$$

Based on the value of τ_d and R, the parameters of the PID controller is given in Table 6.2.

6.5.2 The Frequency Response Method

First, we connect the controller with the plant in a unity feedback arrangement, and the proportional gain is set to be very small. Note that we have no knowledge of the transfer function of the plant $G(s)$. Then a short pulse is applied to test the system. We gradually increase the proportional gain K_P until the step response of the closed-loop system shows the behavior of

Controller Type	Controller Parameters
P	$K_P = \frac{1}{RL}$
PI	$K_P = \frac{0.9}{RL}$, $T_I = \frac{L}{0.3}$
PID	$K_P = \frac{1.2}{RL}$, $T_I = 2L$. $T_D = 0.5L$

Table 6.2: Ziegler-Nichols Tuning

undamped oscillation. The minimum gain K_P to produce the undamped oscillation behavior is called the ultimate gain denoted by K_u, and the period of oscillation is called the ultimate period denoted by P_u. The ultimate period can be obtained by the time between successive oscillation peaks. When performing such a test on a process loop, it is important to ensure the oscillation peaks do not reach the limits of the saturation of the actuator. In other words, in order for the oscillation to accurately reveal the process characteristics of the amplitude and frequency when the closed-loop system is neutrally stable, the oscillations must be naturally limited and not artificially limited by the saturation of the actuator.

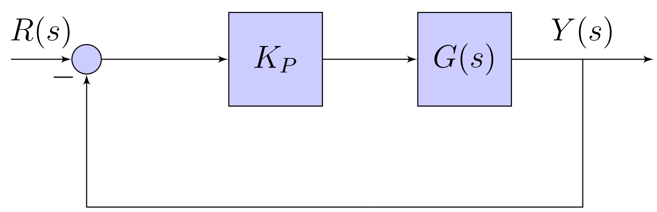


Figure 6.6: Determination of ultimate gain and period

The suggested values of the PID controller are given in Table 6.3.

Controller Type	Controller Parameters
P	$K_P = 0.5K_u$
PI	$K_P = 0.45K_u$, $T_I = \frac{P_u}{1.2}$
PID	$K_P = 1.6K_u$, $T_I = 0.5P_u$. $T_D = 0.125P_u$

Table 6.3: Ziegler-Nichols Tuning

Part III

Computer Aid Tools

Chapter 7

Matlab

MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and proprietary programming language developed by MathWorks. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, C#, Java, Fortran and Python. By its nature, this note cannot cover all the background and details necessary for a complete understanding of MATLAB. It should give you enough information to be able to apply MATLAB to the analysis and design of control systems. For further details, you are referred to other sources, including MATLAB reference manuals.

As you work in MATLAB, you issue commands that create variables and call functions. For example, create a matrix variable named A by typing this statement in at the command line:

```
A = [1 2; 4 6]
```

7.1 Control System Toolbox

Control System Toolbox provides algorithms and apps for systematically analyzing, designing, and tuning linear control systems. You can specify a system as a transfer function, zero-pole-gain. Apps and functions, such as step response plot, let you analyze and visualize system behavior in the time domain. You can tune PID controllers. You can validate your design by verifying rise, overshoot, settling time and other requirements.

7.1.1 Test Signals

```
t = (-1:0.01:1)';
unitstep = t≥0;
ramp = t.*unitstep;
quad = t.^2.*unitstep/2;
plot(t,[unitstep ramp quad])
```

7.1.2 Model Interconnections

The control system toolbox provides functions to construct the transfer function of a complex system by using the interconnection of simple subsystems. The concept is similar to the block diagram reduction.

```
H1 = tf(2,[1 3 0])
H2 = zpk([],-5,5)
% Series Connection
H = H2 * H1
% or equivalently
H = series(H1,H2);
% Parallel Connection
H = H1 + H2
% or equivalently
H = parallel(H1,H2);
% Feedback Connections
H = feedback(H1,H2)
%Note that feedback assumes negative feedback by default. ...
    To apply positive feedback, use the following syntax:
H = feedback(H1,H2,+1);
```

7.1.3 PID Tuner

```
s=tf('s');
sysp=1/(600*s^2+70*s+1);
systd = set(sysp,'InputDelay', 5);
```

```
pidTuner(systd,'P')
```

The command pidTuner(sys,type) launches the PID Tuner app and designs a controller of type type for plant sys. The PID Tuner app is shown in Fig. 7.1. In the app, the type of the controller can be changed. Sliding the two bars to adjust the response time from slower to faster and transient behavior from aggressive to robust.

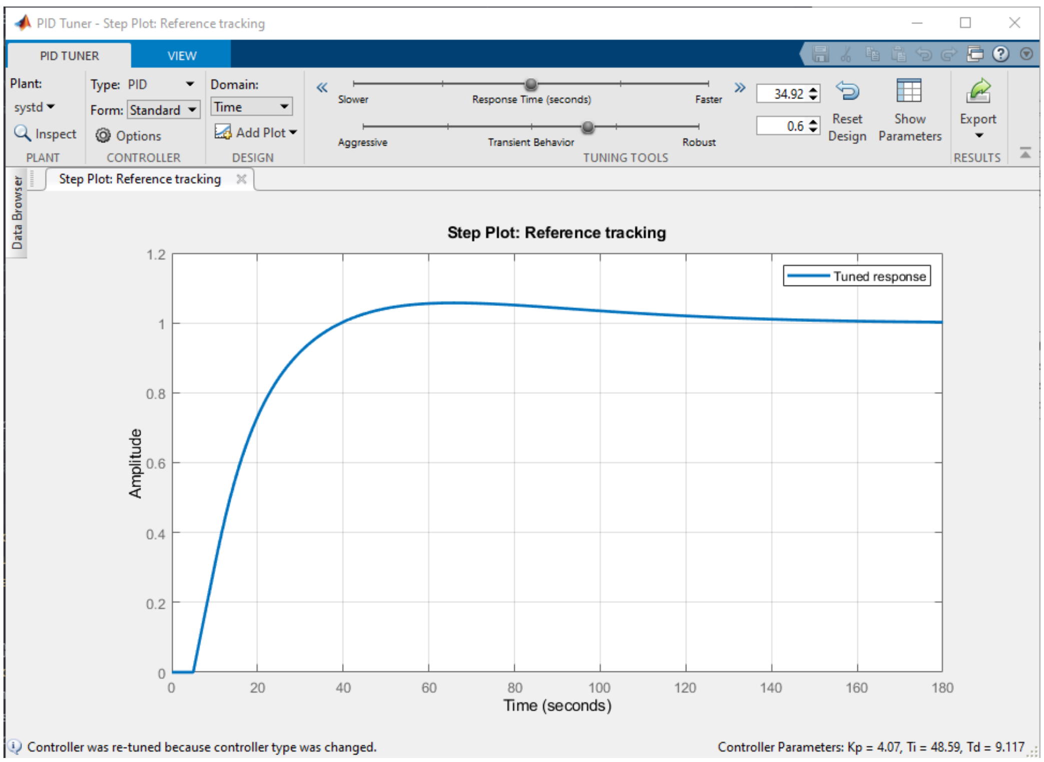


Figure 7.1: PID Tuner App

7.2 Symbolic Math Toolbox

Symbolic Math Toolbox provides functions for calculus of polynomials since transfer functions can be written as the ratio of two polynomials.

```
% Symbolic Toolbox
```

```
syms s m1 m2 b ks kw R
% Polynomials
f1 = s^2+s*b/m1+ks/m1+kw/m1;
f2 = -s*b/m1-ks/m1;
f3 = -s*b/m2-ks/m2;
f4 = s^2+ s*b/m2+ks/m2
f5 = kw*R/m1;

%Multiplication of polynomials
den = collect(f1*f4-f2*f3,s)
den = simplify(den)
den = collect(den,s)

num = collect(-f5*f3,s)
```

Symbolic Math Toolbox also provides functions for matrix calculus.

```
% Define real variables
syms h r real

% Define matrix Variables
P = sym('P', [3 3],'real'); % symbolic real matrix

% Define symmetric matrix variables
P = triu(P,0) + triu(P,1).'; % make P symmetric

Q = sym('Q', [3 3],'real');
S = sym('S', [3 1],'real');

A = [1 h h^2/2; 0 1 h; 0 0 1];
Gamma = eye(3);
C = [1 0 0];

K = Gamma*S/r;

Pk_k_1 = (A-K*C)*P*(A-K*C)'+Gamma*Q*Gamma'-K*r*K';

a = expand(Pk_k_1(1,1));

pretty(a);
```

Chapter 8

LabView

Laboratory Virtual Instrument Engineering Workbench (LabVIEW) is a system-design platform and development environment for a visual programming language from National Instruments. LabVIEW is systems engineering software for applications that require test, measurement, and control with rapid access to hardware and data insights. LabVIEW offers a graphical programming approach that helps you visualize every aspect of your application, including hardware configuration, measurement data, and debugging. This visualization makes it simple to integrate measurement hardware from any vendor, represent complex logic on the diagram, develop data analysis algorithms, and design custom engineering user interfaces.

8.1 Getting Started Modules

Begin here http://www.learnni.com/getting-started/ to learn about the program and how to get started.

8.2 MathScript

Launch NI LabVIEW and click Tools → MathScript Window... as shown in Fig. 8.1.

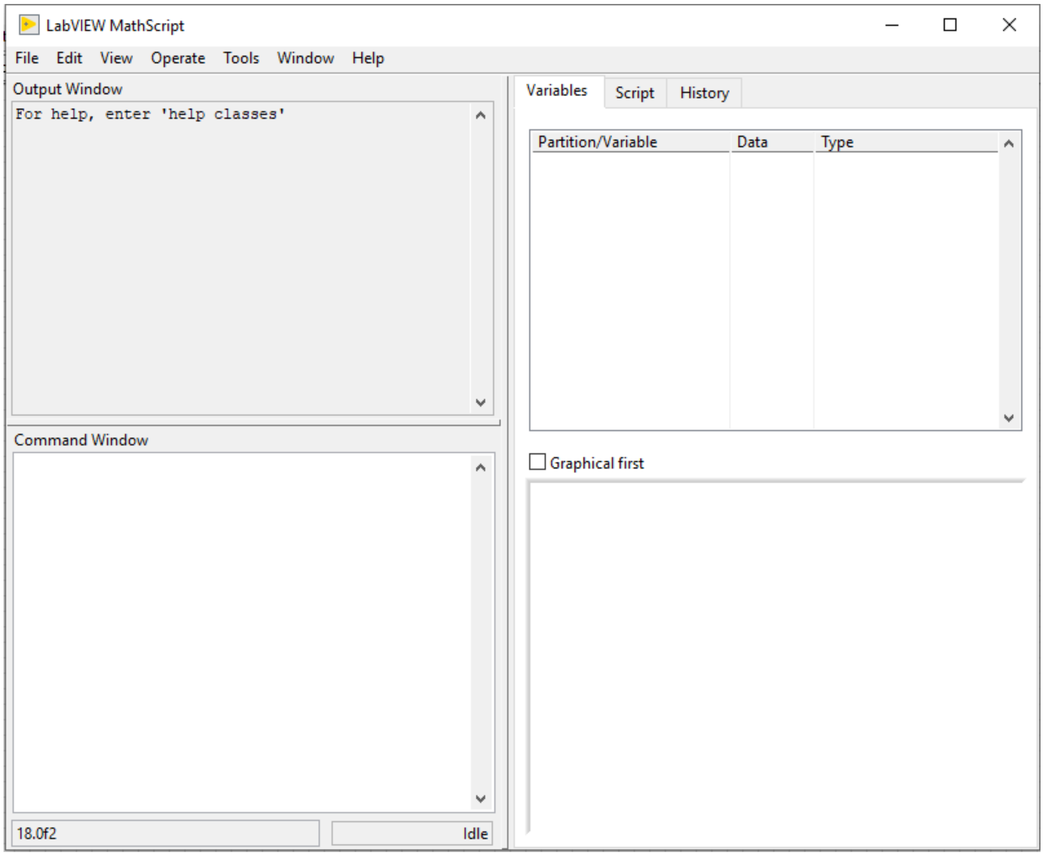


Figure 8.1: MathScript RT Module Functions

8.3 Mathematics

To access the Functions Palette, right click in any blank space of the Block Diagram window. Choose the category Mathematics, and select Polynomial. The Mathematics Palette is shown in Fig. 8.2.

8.4 Control and Simulation

To access the Functions Palette, right click in any blank space of the Block Diagram window. Choose the category Control & Simulation, and select Control Design or Simulation. The Control & Simulation Palette is shown in Fig. 8.3. Click Control Design, choose Model Construction to construct

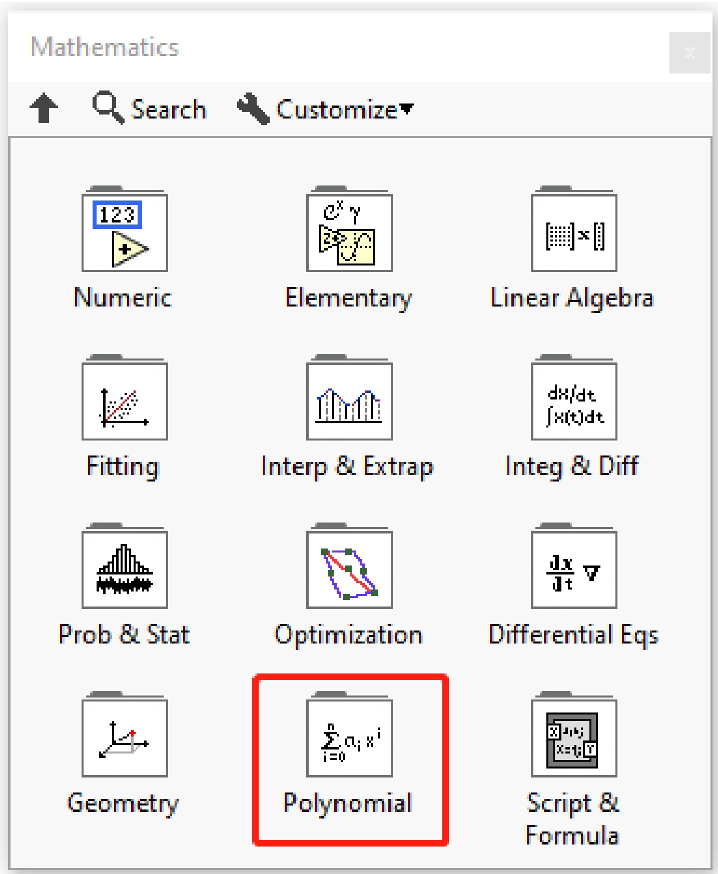


Figure 8.2: Mathematics Functions

transfer functions and Model Interconnection to display the results of the mathematical operations in Fig. 8.4.

8.5 Examples:

8.5.1 System Modeling

CDEx Nonlinear and Linear Simulation.vi

SimEx Nonliear and Linear Pendulum Simulation.vi

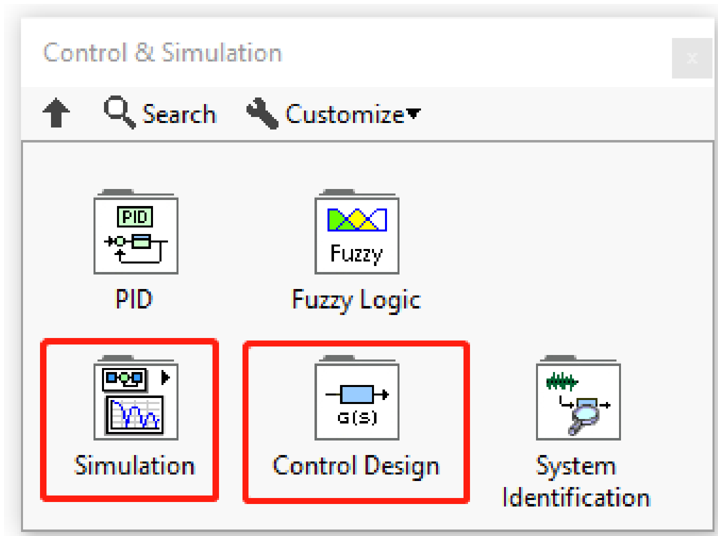


Figure 8.3: Control and Simulation Functions

8.5.2 Block Diagram Reduction

CDEx Block Diagram Reduction.vi

8.5.3 Dynamic Response

On-Off Controller.vi

Aircraft bank angle.vi

8.5.4 System Type

Objective

Get started with control system analysis and design, and get familiar with key control concepts, such as closed-loop control, step response, ramp response, transient response and steady-state error.

Software

LabVIEW and the LabVIEW Control Design and Simulation Module.

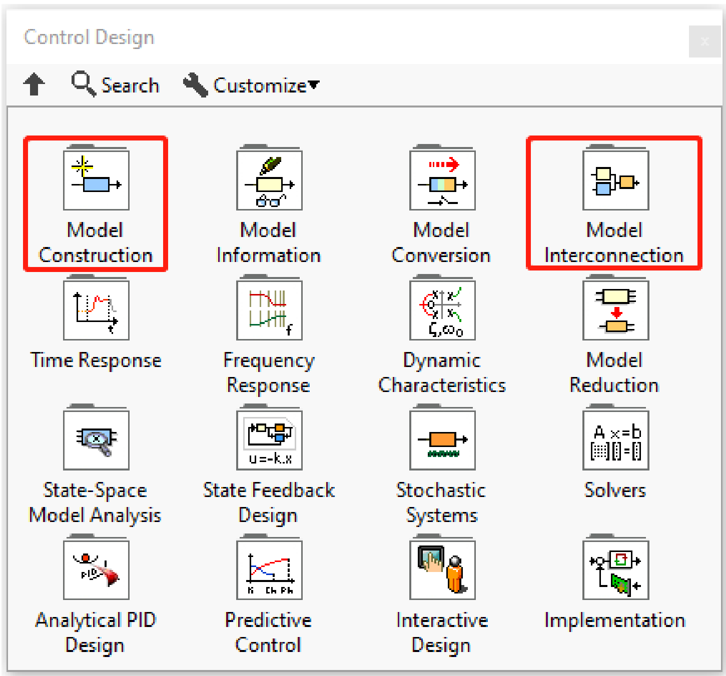


Figure 8.4: Control Design

Lab

1. Launch NI LabVIEW and click Help → Find Examples...
2. In the NI Example Finder window, double-click Toolkits and Modules ⇒ Control and Simulation ⇒ Control Design ⇒ Time Analysis ⇒ CDEx Effect of Controller Type.vi
3. Run the VI continuously.

Report

1. Observe the transient response and record Steady-state Tracking Error and Steady-state Velocity Error when the Control Gain is 0.838 and the Controller Type is Type 1 - one integrator.
2. Repeat Step 1 by changing the type of Controller to Type 0 - No integrators and Type 2 - two integrators.

3. Repeat Step 1 with high gains by moving the slider Controller Gain to the right and with low gains by moving the slider Controller Gain to the left.

4. Describe how the type of control and the Controller Gain affect the step (Steady-state Tracking Error) and the ramp (Steady-state Velocity Error) response.

8.5.5 PID Design

CDEx PID Design.vi

Appendices

Appendix A

Image Credits

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1.5	CC BY-SA 3.0	M.Minderhoud	Adaptive Cruise Control
3.2	CC BY-SA 3.0	Pflodo	Kirchhoff's circuit laws
3.3	CC BY-SA 3.0	Kwinkunks	Kirchhoff's circuit laws
3.4	CC BY-SA 3.0	Ong saluri	Operational amplifier
3.5	CC BY-SA 3.0	Ong saluri	Operational amplifier
4.2	CC BY-SA 3.0	UlrichHeither	Quadcopter

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